The Macroeconomic Consequences of Remittances

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Abstract: We study the impact of remittances on a small open economy with a stochastic limited participation model with cash in advance constraints and costly adjustment of cash holdings. We examine the impact of remittances on the steady state, as well as on the dynamics of the main macroeconomic aggregates. We find that a positive remittances shock forces the exchange rate to depreciate and lowers both output and the interest rate in the period of the shock, irrespective of the proportion of remittances as a percentage of GDP in the economy, but increase output in the subsequent periods, while consumption rises on impact.

Keywords: Migration; Remittances; Limited participation model; Overshooting; Liquidity Effect; Uncovered interest rate parity.

JEL Classification: E40; F22; J61; O15
1 Introduction

Remittances have been on the rise for the last several decades. International estimates of official remittances flows suggest that the total amount of remittances received by developing countries has reached 251 billion U.S. dollars in 2007, up by 118 percent from 2002 (World Bank’s Global Economic Prospects). Moreover, remittances constitute a significant share of some countries’ gross domestic product (Neyapti (2004) and Heilman (2006)). The apparent increase in remittances may in part be attributed to the rapid growth of money transfer institutions, making the money flows more visible, and decreases in the average transaction cost of making remittances. However, the increase in measured remittances is also indicative of an actual increase in these monetary flows.

Remittances gain their significance not just from their size but from the effects of these money flows on the economy. Remittances affect labor market decisions, school retention levels, export sector competitiveness, and they create moral hazard problems (Funkhouser (1992), Glytsos (2002), Edwards and Ureta (2003), Amuedo-Dorantes and Pozo (2004) and Chami et. al. (2005)).

The increasing importance and visibility of monetary remittances has led to an interest in studying the effects of remittances. For several developing countries total remittances already exceed foreign aid and compete in size with foreign direct investment (Connell and Brown (2004), De Haas (2006), Heilmann (2006) and Chami et. al. (2006)). While foreign direct investment (FDI) flows are assumed to be profit driven and therefore considered as a source of development, remittances also have the potential to promote economic growth through increased domestic demand.
Remittances may be motivated by many factors, such as altruism or self-interest (Lucas and Stark (1985)). Consequently, the principal motivation behind remittances may have important implications for the effect of remittances on output in the recipient country. Some researchers believe that altruistically motivated remittances are countercyclical with domestic output; others consider remittances as procyclical with domestic output when they are mainly motivated by self-interest.

Figure 1 illustrates the increasing importance of remittances in certain Latin American countries, comparing remittances and FDI as shares of GDP. Remittances have surpassed FDI in magnitude starting in about 1999, and remittances have been growing while FDI is shrinking. While FDI, and other capital flows, have been volatile and dependent on the economic performance of the receiving countries and region, remittances have been more stable, increasing at a fairly steady pace. ¹

FIGURE 1 ABOUT HERE

Most of the literature on remittances focuses on the microeconomic implication. The literature on the macroeconomic impact of remittances on the recipient country is sparse. This paper explores the impact of remittance flows on output, consumption, interest and exchange rates in the recipient country. We model remittances in a small open economy and analyze the impact of shocks to remittances. We expand a limited participation model that requires that money balances be held to finance certain types of purchases, and that specifies agents incur a cost of adjusting money holdings. These two requirements generate a large and persistent liquidity effect consistent with the stylized facts (Hairault et. al. (2004)).

The main contribution of this paper is to provide a model to examine the impact of a remittances shock on the main macroeconomic aggregates of a small open economy. We also

¹ Refer to Table A1 in the Appendix for the country specific behavior of remittances.
examine the importance of how remittances enter the economy, whether as cash for use directly in consumption or through the financial system. This provides information to domestic governments that are trying to direct a portion of remittances towards investment. We distinguish between the direct effect of remittances on output through investment and the indirect effect through consumption and its multiplier effects. Being able to distinguish the end use of remittances is crucial in looking at the final effect on output (Burgess and Haksar (2005), Heilmann (2006) and Sayan (2006)).

The paper is organized as follows. Section 2 presents a brief summary of the literature review. Section 3 formulates a theoretical model and Section 4 discusses the results. Section 5 provides a robustness check, where the distribution of remittances between consumption and investment, the magnitude of the intertemporal elasticity of substitution, and the amount of time spent working are examined, and where remittances are modeled as procyclical, and where remittance’s shocks are allowed to affect the money growth. Section 6 summarizes and concludes.

2 Literature Review

Residents of labor exporting countries receive substantial annual flows of remittances. Countries like India and Mexico received documented remittances of more than 25 billion U.S. dollars in 2007² (IMF Balance of Payments Yearbook). World Bank figures for 2007 show that remittances were almost 7% of GDP in Ecuador, above 10% in Guatemala, over 18% in El Salvador, and approaching 25% in Honduras. Even in larger economies such as Mexico remittances approached 2.8% of GDP by 2007.

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² Several researchers believe that undocumented remittances are twice the recorded amounts. Refer to Freeman (2006) for more details.
Durand *et. al.* (1996) argue that remittances stimulate economic activity both directly through investment and indirectly through consumption. Even if a large percentage of remittances are used for private consumption, the investment portion may play a significant role in the economy. Furthermore, the use of remittances for consumption stimulates the demand for goods and services in the receiving country, leading to increases in production and employment.

Widgren and Martin (2002) include remittances with FDI and foreign aid as possible sources of accelerating economic growth, although they warn about the nature of remittances. Remittances are not profit driven and are often thought to be intended to mitigate the burden of poor economic performance on the local recipients, thus alleviating consumption but not enhancing growth. ³

Heilmann (2006) argues that remittances constitute a direct transfer of ownership between two individuals with the objective to increase the recipients’ disposable income, but they are difficult to supervise by the receiving country’s government because they are part of informal networks. Heilmann outlines the possibility of remittances promoting a sustainable level of development through its impact on education, health, and consumption but also warns of potential inflation due to stimulation of internal demand for imports.

Chami *et. al.* (2006) develop a stochastic dynamic general equilibrium model to study the implication of remittances on monetary and fiscal policies in the recipient country. They explore the behavior of a subset of real and nominal variables in remittance-dependent economies and in economies where remittances are not significant. In the economy without remittances, the authors show that the optimal monetary policy follows the Friedman rule.

³ Chami *et. al.* (2005) suggest that remittances are compensatory in nature, and document a negative correlation between remittances and GDP growth.
However, in a remittances dependent economy, the optimal monetary policy deviates from this rule and the government resorts to the use of inflation tax. Finally, the authors suggest that remittances insulate households from distortionary government policies.

The literature seems to present two opposing positions concerning the effects of remittances on the economy of the receiving country (Keely and Tran (1989), León-Ledesma and Piracha (2004) and De Haas (2006)). On the one hand, remittances increase the standard of living of receiving households. These funds are spent on consumption, health and education, even finding their way into productive investment. On the other hand, remittances reduce work participation and are rarely invested in productive projects. Remittances increase dependency and may increase economic instability. They increase local demand and put inflationary pressure on prices.

In the rest of this paper we develop and analyze a theoretical model in which remittances transfer resources from the rest of the world to households in a small open economy. Households react as optimizing agents, increasing consumption, leisure, and bond holdings in the steady state. We model remittances as occurring in foreign currency, although the exact form of remittances is not crucial since goods are readily convertible into currency, local or foreign, and vice versa. Our small open economy focus allows us to rationalize our (implicit) assumption that remittances do not impact the remitting economy. Our model generates the expected effects of remittances on optimizing agents, and our goal is to study the quantitative and qualitative dynamic responses that lead to the steady state

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4 Djajić (1998) show that remittances can also increase the welfare of all residents in the labor exporting countries not just those receiving positive amount of remittances.

5 This skirts issues related to the transfer problem discussed in the trade literature, which we do not address. Samuelson (1954) provided a classic analysis of the transfer problem, showing that in a perfectly competitive two-country two-good world, the donor always has reduced welfare and the recipient increased welfare. The donor country’s terms of trade may deteriorate if and only if the donor country’s marginal propensity to consume its own exports is lower than the recipient country’s marginal propensity to consumer the donor country’s exports.
results or that occur in response to shocks to remittances – and to the money supply and technology for consistency.

3 Theoretical Model

We adopt a limited participation model that requires money balances be held to finance certain types of purchases, and agents incur an adjustment cost when altering their money holdings. This model has been used to rationalize a large and persistent liquidity effect. We assume that any monetary shock occurs after households have decided on their money balances, both cash and deposits. This will generate a liquidity effect. To make this liquidity effect persistent we introduce an adjustment cost on cash money holdings, $M^c_t$.

We model the cost of changing money holdings similarly to Hairault et al. (2004), who take into account the time spent on reorganizing the flow of funds. The adjustment cost is a time cost – a reduction in leisure in order to spend time adjusting money balances. The adjustment cost equation is:

$$\Omega_t = \frac{\xi}{2} \left( \frac{M^c_{t+1}}{M^c_t} - \theta \right)^2$$  

(1)

The long run value of $\frac{M^c_{t+1}}{M^c_t}$ is equal to the growth rate of money, represented by the parameter $\theta$, so both the level of $\Omega_t$ and its derivative with respect to $\frac{M^c_{t+1}}{M^c_t}$ is zero in the steady state. The cost of changing $M^c_t$ is an increasing function of the parameter $\xi$, and this parameter allows us to calibrate the size and persistence of the liquidity effect.

The cost of adjusting money holdings implies that bank deposits would not change significantly following a monetary shock, and consequently, the firm will have more funds to absorb as the decrease in the interest rate is stronger and more persistent. In addition, given a
flexible exchange rate regime and uncovered interest rate parity (UIP), this large and persistent fall in the interest rate differential generates an overshooting in the exchange rate in accord with the stylized facts.

**Timing of decisions**

Our small open economy includes a representative consumer-household, a goods-producing firm, a central bank, and a financial intermediary. We have a market for goods, labor, loanable funds, foreign assets, and a money market. Within each period the timing of decisions follows these five stages:

- At the end of period \( t - 1 \) the representative household decides the distribution of money balances, amount of deposits \( (M_t^b) \) and cash \( (M_t^c) \), to be carried into the next period. Note that \( M_t = M_t^c + M_t^b \).

- At the beginning of period \( t \), migrants living abroad remit funds to agents in the small country. After observing the remittances flow, the Central Bank makes its monetary policy decision, choosing the level of monetary injection.\(^6\)

- The credit market then opens. Bank deposits are available in quantity \( M_t^b \) and the firm determines its demand for capital and labor to produce an internationally identical good. The firm borrows from the financial intermediary to finance the needed investment for production.

- The perfectly competitive goods market then opens. Production occurs, and purchasing decisions are made.

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\(^6\) We will specify remittances to adjust for output movements in the receiving country and for price level movements in the receiving country, so that remitters want to transfer real goods or real purchasing power to the receiving country. We specify remittance flows as in the currency of the remitting country.
• At the end of period $t$, the foreign asset market opens. The representative household makes its decision to purchase or sell foreign assets, with returns given by the exogenous world interest rate. Labor is paid at this stage, and firms pay off their intra-period loans to the financial intermediary. As household owns the bank and the firm, household receive dividend payments from the bank and firm as part of household income.

3.1. Structure of the model

The goods market is characterized by perfect competition, with domestic firms and the rest of the world producing an identical good whose price in domestic currency is given by $P_t$. The law of one price holds. Letting $s_t$ denote the price of foreign currency in terms of domestic currency, and keeping in mind that the small open economy assumption implies that the price of the good in foreign currency ($P^*$) and the foreign interest rate ($i^*$) are exogenous, purchasing power parity is given by:

$$P_t = s_t P^*$$ (2)

3.1.1. The household

The representative agent’s objective is to choose a path for consumption and asset holdings to maximize

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$ (3)

where $C$ is real consumption and $L$ is leisure hours. We normalize the time endowment to unity, so leisure is given by

$$L_t = 1 - H_t - \Omega_t,$$

where $H$ is worked hours and $\Omega$ is time spent adjusting money balances.
We specify a parametric constant elasticity of substitution (CES) per-period utility function to facilitate calibration of our model:

\[
U(C_t, L_t) = \left( \frac{C_t^{\gamma} L_t^{\gamma-1}}{1-\sigma} \right)^{1-\sigma}
\]

(4)

Here \( \gamma \) is the relative weight of leisure in the above utility function and \( \sigma \) define the inverse of the intertemporal elasticity of substitution with \( \sigma > 0 \) and \( 0 < \gamma < 1 \).

When the goods market opens – in the fourth stage – the cash-in-advance (CIA) constraint takes the form:

\[
P_t C_t \leq M_t^c + \phi_t R_t
\]

(5)

where \( M_t^c \) denotes cash brought forward from period \( t-1 \). With \( R_t \) being remittances in foreign currency (i.e. dollars) and \( s_t \) being the nominal exchange rate (i.e. pesos per dollar), then \( s_t R_t \) are nominal remittances in domestic currency received by the household. The parameter \( \phi \) take values between 0 and 1, and indicates the percentage of remittances immediately available for consumption (as opposed to being held as bank deposits and only available for consumption in future periods).\(^7\) These parameters allow us to change the channel in which remittances affect the economy, and to see how the end use matters.

Household can hold foreign assets that yield a risk-free exogenous nominal interest rate \( i^* \). In each period the household buys foreign assets \( B_{t+1} \) denominated in the foreign currency, so the nominal exchange rate becomes a key variable in the portfolio decision.

The household budget constraint is given by:

\[
M_{t+1}^c + M_t^b + s_t B_{t+1} + P_t C_t \leq M_t^c + \phi_t R_t + P_t w_t H_t + (1 + i_t)M_t^b
\]

\(^7\) We introduce \( \phi \) to allow us to study policies that induce (force) agents to keep a certain amount of remittances as deposits (increasing funds available for investment).
At time $t$ the household determines consumption $C_t$ and labor supply $H_t$, as well as the amount of money deposited in banks, $M_{t+1}^b$, the amount of money kept as cash, $M_{t+1}^c$, and the foreign asset position $B_{t+1}$. Household income is determined by the real wage $w_t$ and the profits (or dividends) received at the end of the period from both the firm and the bank, $D_t^f$ and $D_t^b$, as well as interest on deposits and on foreign bonds.

The household’s maximization problem can be represented by the value function

$$V(M_t^c, M_t^b, B_t) = \max_{c_t, h_t, M_{t+1}^b, M_{t+1}^c, B_{t+1}} \left\{ U(C_t, 1 - H_t - \Omega_t) + \beta \mathbb{E} V(M_{t+1}^c, M_{t+1}^b, B_{t+1}) \right\}$$

subject to the cash-in-advance constraint (5) and the budget constraint (6). Letting $\lambda_t$ denote the Lagrangian multiplier associated with the budget constraint, the first order conditions for the household’s choice of consumption, labor, money deposits, money-cash holdings, and foreign assets provide the following relationships:

$$\lambda_t = \beta \mathbb{E} [(1 + i_t^*) \lambda_{t+1}]$$

$$-U_h = w_t P_t \lambda_t$$

$$s_t \lambda_t + \beta \mathbb{E} [s_{t+1} (1 + i_t^*) \lambda_{t+1}]$$

$$P_t w_t \lambda_t \frac{\xi_{t+1}}{M_t^c} \left( \frac{M_{t+1}^c}{M_t^c} - \theta \right) + \lambda_t = \beta \mathbb{E} \left[ \frac{U_{t+1}}{P_{t+1}} \right]$$

$$+ \beta \mathbb{E} \left[ P_{t+1} w_{t+1} \lambda_{t+1} \frac{\xi_{t+2}}{M_{t+1}^c} \left( \frac{M_{t+2}^c}{(M_{t+1}^c)^2} - \theta \right) \right]$$

Equation (7) requires equality between the costs and benefits of bank deposits, while equation (8) requires equality between the marginal disutility of working and the marginal benefit – the real wage multiplied by the Lagrange multiplier. Equation (9) requires equality
of the current marginal cost of buying foreign assets (in terms of wealth) with the gains in the
following period from holding such assets today, and equation (10) equates the costs and
benefits related to the choice made at time $t$ of money holdings available for consumption in
the following period. It is clear that if the adjustment cost is zero ($\xi = 0$) then equation (10)
will just equate the household’s cost of holding money in the current period to the marginal
utility of consumption in the following period, properly discounted. However, when
adjustment costs exist ($\xi \neq 0$), the household will compare the cost of changing money
holdings (cash) today to the benefits accrued in the next period with respect to the purchasing
power of money holdings and the in-advance time saved rearranging the household portfolio.

3.1.2. The Firm

We specify the firm’s production technology using a parametric, Cobb-Douglas
functional form:

$$Y_t = e^{\gamma} K_t^\alpha H_t^{1-\alpha}$$

Here $\alpha \in [0,1]$ and $K$ is physical capital. The firm’s objective is to maximize the discounted
stream of dividend payments, where we consider the value of this discounted dividend stream
to households. The firm receives its profits at the end of the period, so the firm borrows
funds from the bank to invest in physical capital at the beginning of the period, with the cost
of borrowing given by the nominal interest rate $i_t$. Consequently, the nominal profits of the
firm are given by

$$D_t^f = P_t Y_t - P_t w_t H_t - P_t (1 + i_t) I_t$$

(12)

with investment evolving according to the law of motion of the stock of physical capital,

$$I_t = K_{t+1} - (1 - \delta) K_t$$

(13)

Note that we assume that firms can only borrow for incremental investments, which need to be paid off
completely by the end of the period.
where $\delta$ is the (constant) depreciation rate. The decision about the use of dividends, either payments to households or reinvestment in the firm, is captured by the ratio of the multipliers associated with the budget constraint of the household in the value function (see equation (7)), as it reflects the consumer’s variation in wealth. The value function of the firm is then

$$V(K_t) = \max_{\{H_t, K_{t+1}\}} \{D_t^f + E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \right] V(K_{t+1}) \}$$

(14)

Note that the discount factor $\frac{\beta \lambda_{t+1}}{\lambda_t}$ can be written as $[E_t(1 + i_t)]^{-1}$, reflecting the fact that the appropriate discount rate is time varying and reflects the expected value of the market-determined interest rate.

The first order necessary conditions for the household’s choice of labor and capital take the form:

$$w_t = (1 - \alpha) \frac{Y_t}{H_t}$$

(15)

$$1 + i_t = \beta E_t \left[ \frac{P_{t+1} \lambda_{t+1}}{P_t \lambda_t} \left( \alpha \frac{Y_t}{K_{t-1}} + (1 - \delta)(1 + i_{t+1}) \right) \right]$$

(16)

Equation (15) indicates that the cost of hiring an additional worker should equal that worker’s marginal productivity, and equation (16) requires equality between the cost and benefit of the marginal investment.

3.1.3. The Central Bank

The money stock evolves according to

$$M_{t+1} = M_t + X_t$$

(17)

where the Central Bank’s money injection is defined as

$$X_t = (\theta_t - 1)M_t$$

(18)
and $\theta_t$ represents the monetary growth factor, itself possibly a function of the size of the remittances flow. Equation (17) indicates that money growth in the economy depends on the existing stock of money $M_t$ and the monetary injection implemented by the central bank $X_t$.

The timing here is that $M_t$ is the beginning-of-period $t$ money stock. After remittances occur in period $t$, the central bank decides on the monetary injection, $X_t$, and this injection determines the money stock carried forward into period $t+1$.

The monetary growth factor $\theta_t$ is specified as:

$$\log(\theta_{t+1}) = (1 - \rho_\theta) \log(\overline{\theta}) + \rho_\theta \log(\theta_t) + \varepsilon_{\theta,t+1}$$ (19)

We specify the technology shock to the production function in the usual way,

$$\log(z_{t+1}) = (1 - \rho_z) \log(\overline{z}) + \rho_z \log(z_t) + \varepsilon_{z,t+1}$$ (20)

We also define $g_t$ as the growth factor for remittances, which evolves according to the first order autoregressive process:

$$\log(g_{t+1}) = (1 - \rho_g) \log(\overline{g}) + \rho_g \log(g_t) + \varepsilon_{g,t+1}$$ (21)

Here $\varepsilon_{g,t+1}, \varepsilon_{\theta,t+1}$, and $\varepsilon_{z,t+1}$ are white noise innovations with variance $\sigma_g^2, \sigma_\theta^2$, and $\sigma_z^2$, respectively.

3.1.4. The financial intermediary

At the beginning of the period, the financial intermediary or ‘bank’ receives deposits from the household, $M_t$, receives a portion of remittances as deposits, $(1 - \phi)s_t R_t$, and receives the monetary injection as deposits, $X_t$. These funds are then available for lending to the firm to pay for the firm’s investment in physical capital. At the end of the period, the

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9 The deposit amount from remittances could be zero if the total amount of remittances received is immediately disbursed to the agent such that it will just add to cash available for consumption. The monetary injection $X_t$ is a helicopter drop on banks, which can be lent in the current period $t$, earning interest that is then distributed back to the households at the end of the period.
firm repays its loans, and the bank returns deposits to the household along with the appropriate interest payment.

To make this clearer, the bank’s nominal asset balance is given by

\[ P_t I_t = M_t^b + (1 - \phi)s_t R_t + X_t \]  

(22)

Here \( P_t I_t \) are the loans made to firms and the right hand side lists sources of funds including deposits, a portion of remittances, and the monetary injection.

Bank profits per period are equal to the interest on loans minus interest paid on deposits and on remittances deposited in banks. Note that the monetary injection directly into banks is a subsidy to the bank in that there is no interest expense incurred by the bank on those funds. Note too that we have equality between the loan rate and the deposit rate. Absent monetary injections, the bank earns zero economic profits.

\[ D_t^b = (1 + i_t)P_t I_t - (1 + i_t)M_t^b - (1 + i_t)(1 - \phi)s_t R_t \]  

(23)

Putting both expressions together, profits of the intermediary depend only on the money injection provided by the monetary authority

\[ D_t^b = (1 + i_t)X_t \]  

(24)

3.1.5. Closing the model

To complete the model specification it is worth to note that there is an uncovered interest rate parity condition (UIP) from combining equations (7) and (9):

\[ E_t \left[ \frac{P_{t+1}}{(1 + \pi_{t+1})} \right] = E_t \left[ \frac{s_{t+1}}{(1 + \pi_{t+1})} \right] = E_t \left[ \frac{P_{t+1}}{s_{t+1}} \frac{(1 + i_{t+1})}{(1 + \pi_{t+1})} \right] \]  

(25)

Here \( \pi \) is the net inflation rate at time \( t+1 \). Since we are modeling a small open economy with international assets freely traded, the no-arbitrage condition leads to UIP.

We assume that foreign currency denominated remittances are based on the income of the receiving economy, and further assume that remittances are negatively correlated with
receiving country’s income deviations from the steady state. Thus remittances increase when
the receiving country experiences an economic downturn. Our specification follows Chami
et. al. (2006), and is written as:

\[ R_t = E_t \left[ \mathcal{G} P_t \left( \frac{Y\tau}{Y_t} \right)^\tau e^{\xi_t} \right] \]  

(26)

A special cases is \( \tau = 0 \), so that remittances respond only to the domestic price level and to
the growth rate \( g \). For \( \tau > 0 \), remittances react to the state of the recipient economy, rising
when the state of the economy worsens (countercyclical).

3.2. Equilibrium

The system’s equilibrium is characterized by the set of prices and quantities

\[ \Omega_t^P = \{w_t, i_t, P_t, s_t\}_{t=0}^{\infty} \]

\[ \Omega_t^C = \{C_t, H_t, B_{t+1}, M_{t+1}, M^b_t, R_t\}_{t=0}^{\infty} \]

\[ \Omega_t^Q = \{Y_t, H_t, K_{t+1}\}_{t=0}^{\infty} \]

and the vector of exogenous foreign variables \( \{P^*, i^*\} \). Given these prices and quantities, the
set of quantities \( \Omega^C \) maximizes the household’s expected intertemporal utility subject to (5)
and (6), the set of quantities \( \Omega^Q \) maximizes the profits of the firm subject to (12) and (13),
and the set of prices \( \Omega^P \) ensures that the labor market, the loanable funds market, and the
money market all clear, all while satisfying purchasing power parity.

The household can hold any quantity of foreign assets, subject only to its budget
constraint. From equation (6) and market equilibrium we infer that foreign asset holdings
evolve according to

\[ s_t B_{t+1} - s_t (1 + i^*) B_t = P_t (Y_t - C_t - I_t) + (1 - (1 + i_t)(1 - \phi))s_t R_t \]  

(27)
Equation (27) relates domestic production and absorption to an economy’s foreign asset position, giving the balance of payments equilibrium. If a country’s production is greater than its absorption, that country has a balance of trade surplus and a negative capital account, so its foreign asset holdings will increase when there are no remittances flowing into the country. Of course, the actual equilibrium impact of remittances on future bond holdings depends on its impact on output, consumption, and investment.

The set of equations given by the first order conditions, the market equilibriums, and the laws of motion for physical capital, domestic money supply, foreign assets, and the monetary growth factor constitute a non-linear dynamic stochastic system. The system of equations is presented in the appendix (A.1), and the log-linearized system is solved following Uhlig’s (1997) methodology. To solve this system we calibrate certain basic parameters and find the steady state values of the relevant variables to characterize the long-run equilibrium of the economy.

3.3. Calibration and steady state equilibrium

Our calibration is based in part on Hairault et. al. (2004), supplemented with specific parameters we derive from a sample of countries used in this study: Bolivia, Brazil, Colombia, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Panama, and Peru. A time period in this model is a quarter.

Table 1 lists the values we assign to the basic parameters. The capital share, $\alpha$, is set to 0.4. The subjective discount factor $\beta$ is set at 0.988, implying a real interest rate equal to 1.2% per quarter. The depreciation rate on capital is set to 2% per quarter. We set the parameter $\gamma$ to 0.74, which implies that the representative household devotes 80% of its time endowment to non-working activities, roughly a 34-hour work week. The remaining
parameters are derived from data from our sample of Latin American countries covering the period 1990 to 2004, and then converted into quarterly measures. The data come from the World Bank’s World Development Indicators database. The parameter \( v \) represents the average of the trade balance to GDP, and is used to determine the long-run real debt-to-GDP ratio in our steady state calculation. The long run gross inflation factor is given by \( \Pi \), and is based on the average inflation factor of the countries in our sample. We set the average gross money growth rate parameter, \( \theta \), to 1.038, or 3.8\% per quarter. Remittances are calibrated to have a steady-state growth rate, \( g \), of 5.5\% per quarter. The persistence coefficient of the remittance’s shock, \( \rho_g \), and the standard deviation of the remittance’s innovation, \( \sigma_g \), are obtained from regressions on data for countries in our sample. Similarly, the parameters of the money process, \( \rho_\theta \) and \( \sigma_\theta \), are obtained from regressions. Finally, we calibrated the technology shock, persistence and variance, to standard levels.

**TABLE 1 ABOUT HERE**

We assume the existence of positive adjustment costs to allow for the liquidity effect, and consider the case of a small but positive adjustment cost parameter, \( \xi = 10 \). This positive adjustment costs represent lost time rearranging money cash balances of almost 6 minutes per week.

The equations are written to describe a stationary system and are the ones presented in the beginning of A.1 in the appendix. Nominal variables are made stationary by dividing them by the lagged domestic price level. The main variables are:

\[
m_t = M_t / P_{t-1} ; m^b_t = M^b_t / P_{t-1} ; \pi_t = P_t / P_{t-1} ; b_t = s_t B_t / P_{t-1} ; \Gamma_t = s_t \Re_t / P_{t-1} \]

3.4.1 Steady state equilibrium
We outline the calculation of steady state equilibrium values for the remaining variables in this section. Obviously adjustment costs disappear in the steady state, and steady state values do not need time subscripts. In the long-run equilibrium we assume the domestic gross inflation rate is given by the gross money growth rate so that $\Pi = \theta$.

We look at a steady state in which the domestic and foreign inflation levels are the same, so purchasing power parity implies that the change in the nominal exchange rate is constant. $^{10}$ Consequently the uncovered interest rate parity condition implies that the domestic and the foreign interest rates are equal ($i = i^*$). Finally, combining equations (7) and (10) and, after some manipulation, we have that the domestic nominal interest rate in steady state is

$$i = \frac{\Pi}{\beta} - 1$$

We can derive the steady state level of remittances from equation (26) as

$$\Gamma = \mathcal{S} \Pi s$$

To find the steady state capital/output ratio (denoted $\kappa$) we get, from the stationarity of equation (16):

$$1 + i = \beta \left[ \frac{Y}{K} + (1 - \delta)(1 + i) \right]$$

$$\kappa \equiv \frac{K}{Y} = \frac{\alpha \beta}{1 + i - (1 - \delta)(1 + i) \beta}$$

Then from the production function we can solve for the output/labor ratio

$$\frac{Y}{H} = \kappa^{1 - \alpha}$$

which can be used in equation (15) to solve for the real wage

$^{10}$ Note that this assumption sets the steady-state nominal exchange rate to be constant, allowing a different steady-state foreign inflation rate will make the steady-state exchange rate grow at a constant rate.
Solving for $H$ in equation (8), and substituting $P\lambda$ from equation (10), we solve for the consumption/output ratio

$$\frac{C}{Y} = \frac{w\beta(1-\gamma)}{\Pi\gamma} \left[ \frac{1}{\Pi} - \kappa - \frac{\alpha}{1-\alpha} \right]$$

$TB = Y - C - I + (1 - (1+i)(1-\phi))\frac{\Gamma}{\Pi}$ is the adjusted trade balance. Using the calibration for $v=TB/Y$, we obtain the long-run real debt-to-GDP ratio that is equal to the domestic trade balance as a share of GDP

$$\frac{b}{Y} \left( 1 - \frac{1+i^*}{\Pi} \right) = \frac{TB}{Y} = v$$

This and equation (27), together with the capital/output ratio, allows us to write steady state output as

$$Y = \frac{\left[1 - (1+i)(1-\phi)\delta_s - \frac{w\beta(1-\gamma)}{\Pi\gamma}\right]}{\left[v - 1 - \frac{w\beta(1-\gamma)}{\Pi\gamma} - \frac{\alpha}{1-\alpha} + \delta_k\right]}$$

The steady state physical capital stock will be given by $K = \kappa Y$, and steady state investment by $I = \delta K$.

The steady state stock of foreign assets in real terms is derived from the balance of payments equilibrium (27), so the household’s stock of foreign assets in real terms is

$$b = vY \left[ \frac{1}{1 - \frac{1+i^*}{\Pi}} \right]$$

Consequently, the steady state consumption level is given by:
\[ C = Y + (1 - (1 + i)(1 - \phi)) \frac{\Gamma}{\Pi} - I - \left(1 - \frac{1 + i^*}{\Pi}\right)b \]

Given the definition of real money balances, then its steady state level is:

\[ m = m^b + m^c \]

From the CIA constraint, steady state real money-cash balances are:

\[ m^c = \Pi C - \phi \Gamma \]

Then using (22) and the definition of money, the household’s steady state real money balances are

\[ m = I + C - \frac{\Gamma}{\Pi} \]

From the definition of preferences, and denoting the shadow price associated with household real wealth by \( \Lambda_i = P_i \lambda_i \), then the marginal utility of wealth in the steady state is

\[ \Lambda = \frac{\beta(1 - \gamma)C^{-\gamma - \sigma(1-\gamma)}(1-H)^{\gamma(1-\sigma)}}{\Pi} \]

The steady state values of these variables are presented in Table 2 under three alternative assumptions about the level of remittances, with remittances equal to 1\%, 5\%, and 20\% of GDP. The nominal interest rate is 5.06\% per quarter in either instance, and the capital output ratio is unaffected by the level of remittances. We have the same inflation rate for either level of remittances, as inflation depends only on the steady state money growth rate. Output is affected somewhat by the increase in remittances, and falls by 1.94\% when remittances rise from 1\% to 5\% of GDP and by 15.35\% when remittances rise from 1\% to 20\% of GDP. This occurs because the capital stock and labor hours are also reduced by similar percentages. Meanwhile consumption is higher by about 0.48\% as remittances rise from 1\% to 5\% of GDP and by 3.84\% when remittances rise from 1\% to 20\% of GDP.

TABLE 2 ABOUT HERE
Thus a permanent increase in remittances results in households choosing more leisure and more consumption, and consequently increasing utility. The per-period utility increase that occurs when remittances increase from 1% to 5% of GDP is equivalent to a ceteris-paribus increase in steady state consumption of 1.9% (calculated at the steady state consumption level when remittances are 1% of GDP). Alternatively, the per-period utility increase is equivalent to a ceteris paribus increase in leisure hours of under 0.5%. For the case of remittances increasing from 1% to 20% of GDP, the increase in utility is equivalent to a ceteris paribus 15.7% increase in consumption (relative to steady state consumption when remittances are 1% of GDP), or to a ceteris paribus 3.9% increase in leisure hours (again relative to steady state leisure when remittances are 1% of GDP). Remittances increase household utility, but they both increase consumption and leisure, and hence do not necessarily lead to an increase in steady-state domestic production.

4 Dynamics

Given the steady states values from the previous section, we first analyze the dynamics of the nominal interest rate, output, the nominal exchange rate, and consumption following expansionary shocks to money and technology to verify the properties of the model, and then we examine the impact of remittances shocks to the main macroeconomic aggregates in depth. We present results for the case of small but positive adjustment cost of about 6 minutes per week ($\xi = 10$).

Our model allows a variety of specifications for the percentage of remittances going to consumption and investment. Since the main dynamics can be observed in our baseline specification, with remittances going almost entirely for consumption ($\phi = 0.99$), we present impulse responses only for this case for the sake of brevity and provide a brief discussion of
how different assumptions on the distribution of remittances will affect the impulse response functions. The results presented here hold for an elasticity of substitution of 1.01.

4.1 Monetary Shock

The impulse response functions presented in this section are those following a 3.8% increase in the home money growth factor in period 0, a magnitude large enough to bring money growth to a halt in the case of a negative shock. The case with remittances being 1% of GDP is illustrated with a solid line, the case with remittances being 5% of GDP with dashed lines, and the case with remittances being 20% of GDP with dotted lines.

The limited participation model with adjustment cost enables us to generate the observed impact of a monetary shock on the main macroeconomic aggregates. The monetary injection leads to an instantaneous rise in inflation, but the inability to reallocate deposits within the period put a downward pressure on the nominal interest rate that leads to a drop in the interest rate. The magnitude of the drop in the interest rate is determined by the size of the adjustment cost and the proportion of remittances as a percentage of GDP. This liquidity effect is persistent because firms raise their investment the period of the shock, which increases the capital stock and lowers the marginal product of capital, leading firms to reduce their demand for loans more than the household’s reduction of money deposits the following period. This is shown below in the top-left panel of Figure 2.

FIGURE 2 ABOUT HERE

The output response to a monetary shock depends on the adjustment costs and the magnitude of remittances as a percentage of GDP. An expansionary monetary shock generates a positive wealth effect, which is allocated to increase leisure in the first period because of the cash-in-advance constraint and adjustment cost of money holdings, reducing
output on impact. From the second period onwards the increase in real wages induce agents to increase labor above the initial steady state level in the cases of remittances being at or below 5 percent of GDP, which combined with the surge in capital from the second period onwards explains the sporadic increase in output above steady state levels in the short run. However, when remittances are 20 percent of GDP the fall in working hours is stronger and its recovery does not reach steady state levels, outweighing the surge in investment and leading to a continuous decline in output. This is shown in the top-right panel of Figure 2.

The monetary injection leads to an instantaneous depreciation of the nominal exchange rate, which arises from the reduction on the return on domestic savings that induces households to hold more foreign assets. The overshooting of the nominal exchange rate shown in the bottom-left panel of Figure 2 comes from the uncovered interest rate parity (equation (25)), which requires the interest rate differential to be equal to the expected rate of appreciation, leading to the subsequent appreciation. This overshooting is accentuated by the size of the adjustment costs and the magnitude of remittances as a percentage of GDP, as it creates a larger and persistent liquidity effect that requires a more accentuated appreciation. Even if agents respond to the below-steady-state domestic interest rate with a continuously increase in their holdings of foreign bonds, the initial overshooting of the exchange rate is strong enough to allow for the subsequent appreciation.

The consumption dynamics following the monetary injection are mainly generated by inflationary pressures during the period of the shock. Given that the consumption level is determined by the cash-in-advance constraint, and since the amount on money-cash cannot be changed during the period of the shock, inflation generated by the larger money supply reduces consumption instantaneously, but returns to steady state gradually as the
rearrangement between money-cash and money-deposits is also gradual. Since the main reason for consumption to fall is inflation, which is determined by the size of the monetary shock, and is irrespective of the magnitude of remittances as a percentage of GDP, the response is the same. This is shown in the bottom-right panel in Figure 2.

These results we find here are similar to those obtained in related papers (e.g. Hairault et al. (2004), Chari et al. (2001), and Christiano and Eichenbaum (1992)). Our model also allows us to consider the influence of how we specify the channel by which remittances first impacts the economy. We find that the method by which remittances first enter the economy has a negligible effect on the response of the economy to a monetary shock.

4.2 Technology Shock

We analyze the behavioral response of the main macroeconomic aggregates to a positive 1 percent technology shock maintaining our baseline assumptions: positive adjustment cost ($\zeta = 10$), elasticity of substitution parameter 1.01, and remittances going almost completely into consumption ($\phi = .99$). The impulse response functions for the three cases representing the influence of remittances in the domestic economy are illustrated as in the previous section, and show that the limited participation model generates the effects observed in the literature.

The technology shock has a direct effect on output, which outweighs the fall in inflation to put upward pressure on the nominal interest rate. The lower inflation raises consumption the period of the shock, which fuels an important increase in investment to raise physical capital, exerting pressure to raise the nominal interest rate above its initial steady state level as shown in the top-left panel in Figure 3. The dynamics of the nominal interest rate after the shock are determined by the adjustment of money cash balances. The smooth
increase in money deposits exerts a stronger pressure than the continuous increase in investment to satisfy the above-steady-state consumption level, forcing the nominal interest rate to fall. This downward pressure towards its steady state continues thereafter as investment, inflation, and money deposits returns to their initial steady state levels.

FIGURE 3 ABOUT HERE

The technology shock increases output on impact, irrespective of the adjustment cost or the magnitude of remittances as a percentage of GDP, as shown in the top-right panel in Figure 3. Thereafter, the positive impact on physical capital – fueled by the above-steady-state levels of investment – is reinforced by the increase in hours worked fueled by the rise in real wages. Since these two factors are the main determinants of the production function, their rise results in a permanent increase in output. The positive effect on output is in accord with existing analyses of technological shocks, with its long lasting effect being determined by the continuous investment brought about by the large increase in deposits that outweigh the higher interest rate.

The initial nominal exchange rate response is determined by the rise of the nominal interest rate, which is only partially neutralized by the fall in inflation. The nominal exchange rate appreciates on impact in an overshooting fashion, as shown in the bottom-left panel in Figure 3. This overshooting is governed by the uncovered interest rate parity condition that requires the interest rate differential to equal the expected rate of depreciation. The expected persistent increase in the nominal interest rate \( E_t R_{r+1} > 0 \) generates a positive interest rate differential and thereby causes the persistent expected depreciation of the exchange rate \( E_t \hat{e}_{r+1} - \hat{e}_r > 0 \). From a balance of payments perspective, the above-steady-state domestic interest rate induces agents to reduce their holdings of foreign bonds, forcing the initial
appreciation. As the domestic interest rate returns to its initial level, households start increasing their holdings of foreign bonds, pressuring the nominal exchange rate upwards and producing the slow but continuous depreciation.

The effect of the positive shock to technology on consumption is primarily determined by the cash-in-advance constraint, which in turn is mainly influenced by the inflation dynamics and the flexibility to adjust money balances. In the period of the shock, the predetermined amount of cash and the fall in inflation leads to an increase in consumption (shown in the bottom-right panel in Figure 3). The fact that money cash is brought back to its initial steady state level only slowly allows for above-steady-state levels of consumption to persist in the subsequent periods, returning to the steady state at the same rate as money cash.

4.3 Remittances Shock

We first analyze the behavior of the economy to a 5.5% positive remittances shock in our baseline calibration through its impact on the nominal interest rate, output, nominal exchange rate, and consumption, to then examine the overall effect on the welfare of the receiving economy, measured by the utility and the adjusted trade balance. Note that the magnitude of the shock is large enough to bring the growth of remittances to a halt in the negative case. Our baseline case still assumes that the elasticity of substitution parameter is 1.01, that remittances go almost completely into consumption ($\phi = .99$), and that the adjustment cost on money balances is positive ($\xi = 10$).

4.3.1 Nominal Interest Rate Response

The introduction of a positive remittances shock lowers the interest rate slightly on impact, irrespective of the size of the adjustment cost or the proportion of remittances as a
percentage of GDP. Although the remittances shock increases inflation slightly on the period of the shock for the cases of remittances being at or below 5 percent of GDP, the decrease in investment is relatively larger such that its downward pressure on the interest rate outweighs the upward pressure from inflation. This lower demand for loans exerts the pressure to lower the nominal interest rate below its initial steady state level as shown below in Figure 4. The initial impact on the nominal interest rate is larger when remittances are 20 percent of GDP, as the rise in inflation is at least 5 times larger.

FIGURE 4 ABOUT HERE

The dynamics of the nominal interest rate after the period of the shock are governed by the dynamics of investment and money deposits. For the cases or remittances at or below 5 percent of GDP, the fall in inflation below the steady state, combined with the slower recovery in investment relative to the increase in money deposits in the subsequent period, keeps the interest rate at the lower levels for an additional period before it starts to rise, which itself is mainly due to the slow but continuous rise in inflation back to its steady state. These same dynamics are in play for the case of larger proportion of remittances as a percentage of GDP, but the recovery of investment is much faster than the increase in deposits, prolonging the lower interest rate level for an additional period before starting to rise. Since both investment and money-deposits peak at levels above their initial steady state four quarters after the remittances shock, with similar proportional increases, it is only when inflation starts to rise slowly back to its steady state level that the interest rate begins to rise monotonically back to its original level too, creating a persistent liquidity effect.

4.3.2 Output Response
The remittances shock decreases output on impact irrespective of the existence of adjustment cost or the proportion of remittances in the economy, but its long term dynamics are affected by the magnitude of remittances as a percentage of GDP. When the proportion of remittances as a percentage of GDP is at or below 5 percent the remittances shock slightly lowers the amount of hours worked on impact even if the real wage increases – a wealth effect. Since the capital stock is fixed in the short run, this reduction in labor causes output to fall slightly. However, since labor further declines the following period, outweighing the increase in the capital stock, output decreases for an additional period. This decline in labor is reversed only after two periods, giving rise to an increase in labor that combines with above steady-state capital to produce an increase in output that peaks 10 periods after the shock. It is only then that the decrease in investment to below steady state levels and the slow decline in worked hours force output to fall monotonically.

FIGURE 5 ABOUT HERE

When the proportion of remittances as a percentage of GDP increases to 20 percent we observe a larger decrease in output during the first couple periods, due to dynamics similar to the ones described for the cases of smaller proportion of remittances but accentuated by the fact that households reduce their work hours by a larger proportion even if the increase in real wages is also greater. Investment also recovers at a faster pace, leading to an increase in capital five time larger than in the previous cases and remaining at above steady state levels for additional periods, which together with the partial recovery in labor until the tenth period lead to a greater recovery in output until it peaks. Since the fall in the nominal interest rate is much larger in this case, the increase in investment is also much
larger, resulting in a stronger recovery of output. The volatility of output arising from a remittances shock is exacerbated by the larger proportion of remittances in the economy.

4.3.3 Nominal Exchange Rate Response

The initial exchange rate response to a positive remittances shock is mainly determined by the inflationary pressure, which leads to a proportional depreciation of the exchange rate on impact. In particular, the positive 0.0052 percent deviation from steady-state in inflation is directly translated in a 0.0052 percent depreciation from steady-state in the nominal exchange rate when remittances are 1 percent of GDP, while when remittances increase to 5 percent of GDP, a 0.0161 percent deviation from steady-state in inflation is directly translated in a 0.0161 percent depreciation from steady-state in the nominal exchange rate, and a 0.107 percent deviation from steady-state in inflation is directly translated in a 0.107 percent depreciation from steady-state in the nominal exchange rate for the higher proportion of remittances. This is shown in Figure 6.

FIGURE 6 ABOUT HERE

Note that while subsequent dynamics are determined by the uncovered interest rate parity condition, the rate of appreciation is dependent on the adjustment of deposits. The persistent negative interest rate differential ($E_t R_{t+1} < 0$) arising from the liquidity effect is counterbalanced by the expected appreciation of the nominal exchange rate in this case ($E_t \hat{e}_{t+1} - \hat{e}_t < 0$), given rise to an overshooting of the exchange rate.

The remittances shock induces agents to hold more foreign bonds. The initial fall in the domestic interest rate – which is magnified by the proportion of remittances as a percentage of GDP – forces the exchange rate to depreciate as agents look for a better return and increase their holdings of foreign bonds immediately after the shock. The increase in
foreign bond’s holdings decelerates as the domestic interest rate begins to rise and the nominal exchange rate to appreciate, improving the return on domestic deposits.

4.3.4 Consumption Response

The consumption dynamics following a remittances shock are generated by the increase in purchasing power brought about by such inflows, outweighing the inflationary pressure during the period of the shock. Since remittances are assumed initially to go almost completely for consumption ($\phi = 0.99$), the increase in inflation in the period of the shock and the fall in real money cash are not strong enough to depress the purchasing power brought about by the remittances shock. Consumption rises on impact, but the size of the increase is greatly influenced by the relevance of remittances in the cash-in-advance constraint, increasing as remittances as a percentage of GDP increase. The monotonic return to steady state levels is determined by the subsequent dynamics of money cash balances and remittances, with the inflationary pressure becoming negligible as it approaches steady state levels. Since the return of money cash balances – from below steady state levels – is much more gradual then the return of remittances – from above steady state levels - then consumption declines slowly, as shown in Figure 7.

FIGURE 7 ABOUT HERE

When we increase the level of remittances as a percentage of GDP, the slow but persistent monotonic decrease in consumption is due to the sequential adjustment of money cash and money deposits, and thus determined by the adjustment cost. Since the remittances response to the shock are identical for the three cases, the increase in the proportion of remittances as a percentage of GDP leads to larger drops in money cash balances, and thus forcing monotonic but faster declines in consumption.
The impact of a remittances shock on the main macroeconomic aggregates in our model reflects the effects found in the empirical literature. Giuliano and Ruiz-Arranz (2008) find that remittances provide a solution for liquidity constraints and an alternative way to finance investment, which is reflected here in the liquidity effect and increase in investment, with its consequent increase in physical capital that gives rise to the recovery of output. This last effect on output is also found by Caceres and Saca (2006) for El Salvador, who find an initial decrease in their index of economic activity that is followed by a temporary recovery, and in Osili (2007), who argue that remittances can play a role in the economic development of the receiving country through its effect on capital accumulation, especially when remittances are used for investment in the receiving country. Similarly, Gupta et al (2009) stress that remittances promote financial development and therefore could promote long term economic growth. With respect to the increase in consumption and leisure, most macroeconomic studies only suggest its increase from a remittances shock, but only microeconomic studies are able to quantify it (Keely and Tran (1989, Leon-Ledesma and Piracha (2004) and Des Haas (2006)).

4.3.5 Utility Response

While the impact of a remittances shock on the main macroeconomic aggregates of our small open economy provides an adequate understanding of its effect at the macro level, its overall impact on the welfare of the representative agent is still somewhat elusive. In order to obtain the agent’s welfare gain from a remittances shock, we analyze the utility of the representative agent under our previous cases. Note that, in our benchmark case with remittances set at 1 percent of GDP, steady-state per-period utility is -100.213. In the case of remittances being 5 percent of GDP, steady state per-period utility increases to -100.208, and
in the case of remittances being 20 percent of GDP steady state per period utility further increases to -100.175.

When we introduce the positive 5.5 percent remittances shock to the benchmark economy with remittances being 1 percent of GDP, per-period utility decreases slightly on impact due to the fact that the relatively small increases in consumption and leisure (decrease in worked hours) are not enough to outweigh the introduction of the adjustment cost, which decreases leisure. These dynamics are shown below in Figure 8. When we increase the proportion of remittances to 5 percent of GDP, per-period utility does increases in the period of the shock, from -100.208 to -100.2078. This small but positive increase in utility arises from the fact that the now larger increases in consumption and leisure are enough to outweigh the relatively similar increase in adjustment costs, leading to the observed improvement in utility. As the proportion of remittances as a percentage of GDP gets larger the improvement in utility following a remittances shock also increases. 11

FIGURE 8 ABOUT HERE

While the level of the utility is determined by the proportion of remittances as a percentage of GDP, the utility dynamics following the period of the shock are influenced by the effect of the adjustment cost directly and indirectly. It returns monotonically when there are positive adjustment costs, both because consumption returns to its steady-state monotonically but also because the adjustment cost dissipates slowly through time.

These utility dynamics are of course magnified for the case of remittances being 20 percent of GDP, with the main difference being on the impact on the macroeconomic aggregates. Here the 5.5 percent remittances shock results in almost 1 percent increase in

11 In terms of time, the remittances shock allows the agent to use 0.04 more minutes when there are positive adjustment costs, but this cost implies that she spends 6 minutes rearranging money balances.
consumption and around 0.007 percent decrease in worked hours (increase in leisure), magnitudes that overwhelm the detrimental effect of the adjustment cost on per-period utility, as shown above.

4.3.6 Adjusted Trade Balance Response

We also examine the impact of the positive remittance shock on the adjusted trade balance (since we are including remittances to domestic production to then subtract domestic absorption). Figure 9 illustrates these trade balance dynamics, with the impulse response functions showing that a remittances shock has a positive impact on the trade balance in the short run, having a larger effect when the proportion of remittances as a percentage of GDP increases.

FIGURE 9 ABOUT HERE

In the calibration with remittances being 1 percent of GDP we observe that the trade deficit declines slightly during the period of the shock by 0.005 percent of GDP. These dynamics are determined by the behavior of output and remittances relative to the behavior of consumption and investment, which allows the trade deficit to remain lower for a few periods before starting to deteriorate again. When remittances are calibrated to be 5 percent of GDP this effect gets magnified. In this case the trade deficit falls in the period of the shock by 0.014 percent of GDP, mainly due to the larger contribution of remittances and smaller drop in output relative to the increases in consumption and investment. This behavior in accentuated as we increase the proportion of remittances as a percentage of GDP. Of course, the larger the proportion of remittances as a percentage of GDP the greater the positive contribution on the adjusted trade balance, as the increase in remittances outweighs the increases in consumption and investment by larger percentages.
In terms of its effect on the behavior of foreign bonds, the representative agent increases its holding of next-period foreign bonds in all cases in response to the fall in the domestic nominal interest rate on the period of the shock, leveling off at a slightly higher level than in the beginning since the interest rate doesn’t fully recover. This holds irrespective of the relative weight of remittances in terms of GDP.

5 Robustness of the Remittances Shock

The qualitative dynamic response to a remittances shock in our model is robust to alternative specifications regarding the amount of remittances for consumption and investment. Here we discuss the differences in magnitude of the main dynamics in response to different assumptions regarding the percent of remittances that go to consumption, the magnitude of the intertemporal elasticity of substitution, the amount of time spent working, and the cyclicality of remittances with respect to domestic output. Since the dynamic responses behave in similar fashion, here we use the benchmark calibration with remittances being 1 percent of GDP and with adjustment costs being 6 minutes per week ($\xi = 10$).

Since many governments in remittance-receiving countries are currently exploring policy tools that could direct a portion of remittances towards investment, we begin this section by increasing the proportion of remittances channeled to the financial intermediaries (and thus reducing the proportion available for consumption) and examine the impact of a remittances shock as we allow for its effect to work its way through investment. As we lower the amount of remittances available for consumption from the initial $\phi = .99$ to $\phi = .85$ and then to $\phi = .70$, the fall in the nominal interest rate becomes accentuated, and thus generating a stronger and longer-lasting liquidity effect, as shown below in Figure 11. This stronger liquidity effect also increases investment and therefore capital, generating a stronger and
faster recovery of output as one allows for a greater fraction of remittances to initially go to the bank and thus becoming available for lending and investment. Following the nominal interest rate dynamics, as we lower the amount of remittances available for consumption from $\phi = .99$ to $\phi = .85$ and to $\phi = .70$, the initial depreciation of the nominal exchange rate also becomes accentuated, depreciating by an additional 30 percent from the base case every time the fraction available for consumption falls to a lower level. As expected, consumption’s dynamic response becomes smaller as we reduce the percentage of remittances used to finance consumption, falling by almost 40 percent as we allow for the fraction available for consumption to fall to 70 percent.

FIGURE 10 ABOUT HERE

These dynamics show that as we increase the percentage of remittances devoted for investment will put a downward pressure on the nominal interest rate, as more funds are available for lending, and will consequently decrease consumption too. This lower interest rate will induce agents to adjust their investment portfolio towards foreign bonds, depreciating the domestic currency on its way, and to increase their capital accumulation, thus allowing for a significant improvement in production.

Since the output response to a remittances shock depends on the wealth effect and on its implied effect on worked hours, we also examine the role played by the intertemporal elasticity of substitution. The instantaneous utility function used in this study was originally calibrated such that the relative risk aversion degree parameter was equal to 1.01, implying an intertemporal elasticity of substitution equal to 0.99. We now focus attention on the effect of the remittance’s shock on the main macroeconomic aggregates as we vary $\sigma$, the inverse of the intertemporal elasticity of substitution. It is easy to show that when $\sigma > 1$ consumption
and leisure are complements and that when $\sigma < 1$ consumption and leisure are substitutes. Thus as $\sigma$ increases the household’s willingness to smooth her consumption across time also increases. We allow $\sigma$ to take the values 0.5, 1.01, and 1.5.

Increasing $\sigma$ tends to reduce the demand for loans directed to investment since each household is a shareholder of the firm and is trying to smooth consumption, even if firms should try to take advantage of the temporary fall in the nominal interest rate. This reduces the fall of the nominal interest rate on impact, and even raises it for the larger $\sigma$, and tends to amplify the overall liquidity effect in the long run. This adjustment in the credit market also affects the nominal exchange rate, with a higher $\sigma$ leading to a greater overshooting of the nominal exchange rate.

The output response to a larger $\sigma$ reduces and even reverts the fall in output under the smaller $\sigma$. As shown below in Figure 12, for the cases of $\sigma$ being greater than 1.01 we actually have an increase in output following the remittances shock. These dynamics follow the behavior of worked hours, since the capital stock is predetermined in the period of the shock. The parameter $\sigma$ determines the degree of substitutability between consumption and leisure, the increase in consumption resulting from the remittances shock – also shown above – leads to a decrease in the marginal utility of leisure and consequently to a decrease in leisure time (increase in worked hours) for $\sigma > 1$, and to an increase in the marginal utility of leisure and consequently to an increase in leisure time (decrease in worked hours) for $\sigma < 1$.

FIGURE 11 ABOUT HERE

Increases in the relative risk aversion parameter therefore reduce, and even overturn, the fall in the nominal interest rate and in output, magnifies the overshooting depreciation of the nominal exchange rate, and does not alter the consumption response.
We also allow the amount of worked hours to vary to allow for conflicting views about the right calibration of \( H \) for Latin America. We allow the parameter \( \gamma \) – the relative weight of leisure in the utility function – to vary to examine the cases when the representative agent spends 17 percent of total time working (28.6 hours per week), 20 percent of total time working (33.6 hours per week), and 23 percent of total time working (38.6 hours per week).

Increasing \( H \) reduces the fall of the nominal interest rate on impact, and therefore dampens the liquidity effect in the long run. As the household spends more time working, the positive remittances shock increases the demand for goods but reduces the fall in investment, thus alleviating the downward pressure on the interest rate. This smaller reduction in the domestic interest rate reduces the portfolio adjustment towards foreign bonds, and thus alleviating the initial depreciation of the nominal exchange rate.

FIGURE 12 ABOUT HERE

The output response to a larger \( H \) is determined by the relation between the predetermined capital and labor input. The household earning more for the additional time spent working makes the additional purchasing power brought about the remittances shock less influential in the increase in real wages, and thus in the initial fall of working hours. This causes the fall in output to dampen as \( H \) increase, but also to reduce the subsequent increase in output as capital and worked hours recover. In fact, the greater earnings brought about from the larger amount of time spent working also reduces the impact on consumption, since the 5.5 percent remittances shock is now relatively smaller in the household’s budget constraint. The remittances shock increases consumption, but its effect on consumption is smaller, as shown above in Figure 13. Increases in the work hours parameter therefore
reduces the fall in the nominal interest rate, the overshooting depreciation of the nominal exchange rate, and the increase in consumption, and smoothes the output response.

A related issue in remittances receiving countries, especially since a significant portion of these funds enter the economy through informal channels, is their ability to sterilize such inflow of funds. To this end, we allow for the monetary growth dynamics – equation 19 – to also be affected by the remittances shock such that the monetary growth factor $\theta_t$ is now specified as:

$$
\log(\theta_{t+1}) = (1 - \rho_{\theta}) \log(\bar{\theta}) + \rho_{\theta} \log(\theta_t) + \rho_{\theta, g} \log(g_t) + \varepsilon_{\theta, t+1}
$$

(28)

In our base specification, with remittances being 1 percent of GDP, we allow for the correlation parameter $\rho_{\theta, g}$ to increase from no correlation ($\rho_{\theta, g} = 0$) to positive correlation ($\rho_{\theta, g} = 0.0005$ and $\rho_{\theta, g} = 0.0015$). When one moves from no correlation between the remittances shock and money growth to a 0.05 percent correlation, the 5.5 percent remittances shock magnifies the rise in inflation and the drop in money deposits, but the overwhelming drop in investment creates a greater pressure in the interest rate, making it to fall by approximately 40 percent more. This fall in the interest rate is further accentuated when the correlation is increased to 0.15 percent, as shown below in Figure 13.

FIGURE 13 ABOUT HERE

This accentuated liquidity effect creates the interest rate differential that gives rise to the larger depreciation of the nominal exchange rate observed in the bottom-left response, while the stepper and more permanent fall in working hours, together with the increasingly below steady state physical capital, creates the permanent fall in output that is accentuated as the sterilization becomes more incomplete. This increase in the inability of central banks to fully sterilize the inflow of remittances has a direct impact on the behavior of consumption.
While the remittances shock creates an upward pressure in consumption, the increasing correlation between the remittances shock and money growth gives rise to an increasing inflationary pressure that outweigh the upward pressure on consumption, even reducing consumption as the correlation becomes larger than 0.11 percent.

To conclude this robustness check, we also investigate the effect of a remittances shock on the economy under alternative specifications for the cyclicality of remittances and output - countercyclical or procyclical, thus altering the way in which the remitted funds are spend between consumption and investment, and shedding some light in the altruistic or self-interest motives. We allow the parameter $\tau$ – the relative weight of domestic output on the remittances specification (equation 26) – to change signs to examine the cases when remittances are countercyclical (when $\tau > 0$ and $\phi = 0.3$) and when remittances are procyclical (when $\tau < 0$ and $\phi = 0.7$).

In this case the 5.5 percent remittances shock lowers the nominal interest rate as before, but the liquidity effect is almost doubled in the procyclical case – investment more than doubles but the reduction in money deposits is less than half. Also, while the drop in labor leads to the fall in output under the two cases, its fall is only slightly larger in the procyclical case, but since real wages recover at a faster pace in this case it increases worked hours much quicker, leading to a faster and larger recovery of output. The behavior of the nominal exchange rate is also influenced by the cyclicality of remittances with respect to GDP, as it follows the behavior of inflation and the interest rate, and while for each case the procyclical effect is almost twice as large, the dynamics remain the same as we increase the proportion of remittances as a percentage of GDP.

FIGURE 14 ABOUT HERE
However, when we increase the proportion of remittances as a percentage of GDP we find that consumption continues to rise of impact, with a much smaller increase in the procyclical case due to the reallocation of these funds towards investment, but it then produces a much sharper decline in consumption that peaks 4 periods after the shock below steady state levels. These results show that the macroeconomic response to a remittances shock is independent of the use of funds or remitting motive – countercyclical or procyclicality – which affect the magnitude between different assumptions but not its behavior.

The impact of a remittances shock on the main macroeconomic aggregates in our model corroborates effects found in the empirical literature. Giuliano and Ruiz-Arranz (2008) find that remittances provide a solution for liquidity constraints and an alternative way to finance investment, which is reflected here through the liquidity effect and increase in investment, with its consequent increase in physical capital that gives rise to the recovery of output. This last effect on output is also found by Cáceres and Saca (2006) for El Salvador, who find an initial decrease in their index of economic activity that is followed by a temporary recovery, and in Osili (2007), who argue that remittances can play a role in the economic development of the receiving country through its effect on capital accumulation, especially when such inflows are used for investment. Gupta et al (2009) also stress that remittances promote financial development and therefore could promote long term economic growth.

With regards to the increase in consumption and leisure, only microeconomic studies are able to quantify its increase from a remittances shock (Keely and Tran (1989, Leon-Ledesma and Piracha (2004) and Des Haas (2006)), while macroeconomic studies
only suggest that remittances increase consumption (Ratha (2003), Cáceres and Saca
(2006), Chami et al (2006) and Gupta et al (2009)). Our paper strengthens this link and
shows that a remittance shock increases consumption.

6 Conclusions

Our limited participation model for a small open economy with remittances is able to
capture the behavior of the main macroeconomic aggregates to monetary and technology
shocks, in accord with empirical evidence. The introduction of adjustment costs on money
holdings allows for a persistent liquidity effect, and overshooting of the nominal exchange
rate, in response to monetary innovations.

The novel contribution of this paper comes from its ability to examine the dynamic
response of major macroeconomic aggregates including the nominal interest rate, output, the
nominal exchange rate, and consumption to remittances shocks. We find that a remittances
shock in our model with adjustment costs will lower the nominal interest rate and create a
liquidity effect, reduce output for several periods before it eventually increases above its
initial level, depreciates the nominal exchange rate on impact and then continuously
appreciate, and increases consumption on impact to then smoothly return to its initial steady
state due to the adjustment cost on money balances.

With respect to changes in the proportion of remittances as a percentage of GDP, we
find that as the percentage increases from 1 to 5 percent of GDP both physical capital and
output fall by almost 1.94 percent, as do work hours, while consumption increases by slightly
more than 0.48 percent. When remittances as a percentage of GDP go from 1 to 20 percent of
GDP physical capital and work hours – and consequently output – fall by 15.35 percent, but
consumption increases by 3.84 percent in this case. Thus a permanent increase in remittances
results in households choosing more leisure and allowing for more consumption, increasing per-period utility. Remittances are good for households, but do not necessarily lead to an increase in steady-state domestic production.

We also examine the impact of different modeling assumptions with respect to the end use of remittances, whether they loosen the cash in advance constraint facing households or increase the amount of loanable funds available to financial intermediaries. We find that these alternative specifications affect the dynamic response of the macroeconomic variables to a remittances shock. The decrease in the nominal interest rate and the initial depreciation of the nominal exchange rate are accentuated as the amount of remittances that are available for immediate consumption is reduced. In addition, while a reduction in the percentage of remittances available for consumption will increase the response of output to the remittances shock, this higher level of output is not strong enough to eliminate the fall in consumption brought about by the smaller percentage of remittances available for immediate consumption.

To finish, our model also show that the macroeconomic response to a remittances shock is independent of the use of such remittances or the remitting motive – countercyclical or procyclicality – which affect the magnitude but not its behavior, but that the inability to fully sterilize remittances inflows can exacerbate the fall in the interest rate and the depreciation of the exchange rate, diminishing the recovery of output and limiting the positive impact on consumption. In fact, if the correlation of the remittances shock and money growth is sufficiently larger – as creating a large enough inflationary pressure – consumption can instead fall.
References


Figure 1: Trends of FDI and Remittances for a sample of Latin American Countries

Figure 2: Dynamic response to a 3.8% monetary shock

- 1% ------ 5% …….. 20% of GDP

Figure 2: Dynamic response to a 3.8% monetary shock

- 1% ------ 5% …….. 20% of GDP
Figure 3: Dynamic response to a 1% technology shock

--- 1%  ------ 5%  ...  20% of GDP

Figure 4: Nominal Interest Rate dynamics following a remittances shock

Figure 5: Output dynamics following a remittances shock
Figure 6: Nominal Exchange Rate dynamics following a remittances shock

Figure 7: Consumption dynamics following a remittances shock

Figure 8: Utility dynamics following a remittances shock

Figure 9: Trade Balance dynamics following a remittances shock
Figure 10: Main dynamics for different end uses of remittances when 1%GDP
Dynamic response to a 5.5 percent deviation in remittances shock

--- 1% ---- 15%  ........ 30% for Investment

Figure 11: Main dynamics for different Elasticities of substitution when 1%GDP
Dynamic response to a 5.5 percent deviation in remittances shock

--- 0.5 E.of S. ---- 1.01 E.of S.  ....... 1.5 E.of S.
Figure 12: Main dynamics for different levels of hours worked when 1%GDP Dynamic response to a 5.5 percent deviation in remittances shock

17% ------ 20% …….. 23% Working time

Figure 13: Main dynamics for different levels of correlation when 1%GDP Dynamic response to a 5.5 percent deviation in remittances shock

0% ------ 0.05% …….. 0.15% Correlation
Figure 14: Main dynamics for different assumptions about cyclicality when 1%GDP
Dynamic response to a 5.5 percent deviation in remittances shock
—— 1%  ----- 5%  -------- 20% of GDP
Table 1 – Model Calibration Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<td>0.4</td>
<td>$\gamma$</td>
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<td>$g$</td>
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<td>$\vartheta$</td>
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<td>$\theta$</td>
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Table 2 – Steady State Values

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<tr>
<th>Remittances</th>
<th>1% GDP</th>
<th>5% GDP</th>
<th>20% GDP</th>
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<td>Nominal Interest Rate</td>
<td>0.0506</td>
<td>0.0506</td>
<td>0.0506</td>
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<tr>
<td>Capital/output ratio</td>
<td>11.8439</td>
<td>11.8439</td>
<td>11.8439</td>
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<tr>
<td>Output</td>
<td>1.0402</td>
<td>1.0201</td>
<td>0.8806</td>
</tr>
<tr>
<td>Labor (hours worked)</td>
<td>0.2002</td>
<td>0.1963</td>
<td>0.1695</td>
</tr>
<tr>
<td>Remittances</td>
<td>0.0102</td>
<td>0.0311</td>
<td>0.1765</td>
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<tr>
<td>Capital</td>
<td>12.3201</td>
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<td>10.4294</td>
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<td>Investment</td>
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<td>0.2416</td>
<td>0.2086</td>
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<tr>
<td>Bonds</td>
<td>2.5008</td>
<td>2.4524</td>
<td>2.117</td>
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<tr>
<td>Consumption</td>
<td>0.8339</td>
<td>0.8379</td>
<td>0.8659</td>
</tr>
<tr>
<td>Real Money Balances</td>
<td>1.0705</td>
<td>1.0495</td>
<td>0.9045</td>
</tr>
<tr>
<td>Real Money deposits</td>
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<td>0.2416</td>
<td>0.2086</td>
</tr>
<tr>
<td>Real Money Cash</td>
<td>0.8339</td>
<td>0.8379</td>
<td>0.8659</td>
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<tr>
<td>Inflation</td>
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<td>1.038</td>
<td>1.038</td>
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<tr>
<td>Utility</td>
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<tr>
<td>Trade Balance</td>
<td>-0.03038</td>
<td>-0.02975</td>
<td>-0.02565</td>
</tr>
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Appendix (not for publication)

Table A1: Remittances behavior in countries used in the study

<table>
<thead>
<tr>
<th>Country</th>
<th>Year Range</th>
<th>Percentage</th>
<th>Year Range</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Panama</td>
<td>1980-2006</td>
<td></td>
<td>Peru</td>
<td>1990-2006</td>
</tr>
</tbody>
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A.1. System of Equations

(2) \( \pi_t = \frac{s_t}{s_{t-1}} \pi_t^* \)

(5) \( \pi_t C_t = m_t^c + \phi \Gamma_t \)

(7) \( \Lambda_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} (1 + i_{t+1}) \right] \)

(8) \( -U_t' = w_t \Lambda_t \)

(9) \( \Lambda_t = \beta E_t \left[ \frac{(1 + i^*)}{s_{t+1}} \frac{\Lambda_{t+1}}{\pi_{t+1}} \right] \)

(10) \( w_t \Lambda_t \frac{\pi_t}{m_t} (\Delta M_t^c - \theta) + \Lambda_t = \beta E_t \left[ \frac{U_t'}{\pi_{t+1}} \right] + \beta E_t \left[ w_{t+1} \Lambda_{t+1} \frac{\Delta M_{t+1}^c}{m_{t+1}} (\Delta M_t^c - \theta) \right] \)

(11) \( Y_t = \omega_i K_t^\alpha H_t^{1-\alpha} \)

(13) \( I_t = K_{t+1} - (1 - \delta) K_t \)

(15) \( w_t = (1 - \alpha) \frac{Y_t}{H_t} \)

(16) \( 1 + i_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + i_{t+1}) \right\} \right] \)

(17) \( m_{t+1} = \theta_t \frac{m_t}{\pi_t} \)

(22) \( \pi_t I_t = m_t^b + (\theta_t - 1) m_t + (1 - \phi) \Gamma_t \)

(26) \( \Gamma_t = E_t \left[ \theta(Y^*)^\pi_t \pi_t, Y_t^- e^{\pi t} \right] \)

(27) \( b_{t+1} s_{t-1} (1 + i^*) \frac{b_t}{s_t} = Y_t - C_t - I_t + [1 - (1 + i_t)(1 - \phi)] \frac{\Gamma_t}{\pi_t} \)

Define \( \Delta M_t^c = \frac{M_{t+1}^c}{M_t^c} \), and by previous the definition

(28) \( \Delta M_t^c = \frac{m_{t+1}^c \pi_t}{m_t^c} \)

(29) \( m_t = m_t^b + m_t^c \)

(19) \( \log(\theta_{t+1}) = (1 - \rho) \log(\overline{\theta}) + \rho \log(\theta_t) + \epsilon_{\theta_{t+1}} \)

(20) \( \log(\theta_{t+1}) = (1 - \rho) \log(\overline{\theta}) + \rho \log(\theta_t) + \epsilon_{\theta_{t+1}} \)

(21) \( \log(z_{t+1}) = (1 - \rho) \log(\overline{z}) + \rho \log(z_t) + \epsilon_{z_{t+1}} \)
A.2. The log-linearized system of equations is given by

(2) \[ 0 = -\hat{\pi}_t + \hat{s}_t - \hat{s}_{t-1} \]

(5) \[ 0 = \hat{\pi}_t + \hat{C}_t - \frac{m^c}{C\pi} \hat{m}^c_t - \frac{\Gamma\phi}{C\pi} \hat{\Gamma}_t \]

(7) \[ 0 = E_t \left[ -\hat{\Lambda}_t + \frac{i}{1+i} \hat{t}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} \right] \]

(8) \[ 0 = \hat{w}_t + \hat{\Lambda}_t - (1 - \gamma(1 - \sigma)) \frac{H}{1-H} \hat{H}_t - (1 - \gamma)(1 - \sigma) \hat{C}_t \]

(9) \[ 0 = E_t \left[ -\hat{\Lambda}_t + \hat{\Lambda}_{t+1} + \hat{s}_{t+1} - \hat{s}_t - \hat{\pi}_{t+1} \right] \]

(10) \[ 0 = E_t \left[ -\Lambda \hat{\Lambda}_t - S\beta \hat{\pi}_{t+1} - S\beta \gamma(1 - \sigma) \frac{H}{1-H} \hat{H}_{t+1} - S\beta(\gamma + \sigma(1 - \gamma)) \hat{C}_{t+1} \right. \]

\[ + \beta\pi^2 A w_\xi \left. \frac{1}{m^c} \Delta M^c_{t+1} - \pi^2 A w_\xi \frac{1}{m^c} \Delta M_t \right] \]

where \( S = (1 - \gamma)(1 - H)^{(1 - \sigma)} C^{-\gamma - \sigma(t - \gamma)} \pi^{-1} \)

(11) \[ 0 = -\hat{Y}_t + \alpha \hat{K}_t + (1 - \alpha) \hat{H}_t + \hat{z}_t \]

(13) \[ 0 = \frac{I}{K} \hat{t}_t - \hat{K}_{t+1} + (1 - \delta) \hat{K}_t \]

(15) \[ 0 = -\hat{w}_t + \hat{Y}_t - \hat{H}_t \]

(16) \[ 0 = E_t \left[ \left( \alpha\beta Y K + \beta(1 - \delta)(1 + i) \right) \hat{A}_{t+1} + \left( \beta(1 - \delta)i \right) \hat{s}_{t+1} + \alpha\beta Y \hat{Y}_{t+1} - \alpha\beta \frac{Y}{K} \hat{K}_{t+1} \right. \]

\[ - \left( \alpha\beta \frac{Y}{K} + \beta(1 - \delta)(1 + i) \right) \hat{A}_t - (i) \hat{s}_t \]

(17) \[ 0 = -\hat{m}_{t+1} + \hat{m}_t - \hat{\pi}_t + \hat{\theta}_t \]

(22) \[ 0 = -\hat{\pi}_t - \hat{I}_t + \frac{m^b}{I\pi} \hat{m}^b_t + \frac{m}{I\pi} (\theta - 1) \hat{m}_t + \frac{\Gamma}{I\pi} (1 - \phi) \hat{I}_t \]

(26) \[ 0 = E_t \left[ \hat{m}_t + \hat{\pi}_t - \hat{\theta}_t + \hat{I}_t + \hat{\xi}_t \right] \]

(27) \[ 0 = -\hat{\beta}_{t+1} + \frac{(1 + i^*)}{\pi} \hat{s}_t - \frac{(1 + i^*)}{\pi} \hat{s}_{t-1} + \frac{(1 + i^*)}{\pi} \hat{b}_t + \left( \frac{Y - C - I - b}{b} \right) \hat{\beta}_t + \frac{Y}{b} \hat{Y}_t - \frac{C}{b} \hat{C}_t \]

\[ - \frac{I}{b} \hat{I}_t + \left( \frac{(1 - \phi)(1 + i)\Gamma}{b\pi} \right) \hat{\beta}_t - \frac{(1 - \phi)i\Gamma}{b\pi} \hat{t}_t \]

(28) \[ 0 = -\Delta M_t + \hat{m}^c_{t+1} + \hat{\pi}_t - \hat{m}_t \]

(29) \[ 0 = -(m) \hat{m}_t + (m^b) \hat{m}^b_t + (m^c) \hat{m}^c_t \]

(19) \[ \hat{\theta}_{t+1} = \rho_\theta \hat{\theta}_t + \varepsilon_{\theta_{t+1}} \]

(20) \[ \hat{g}_{t+1} = \rho_g \hat{g}_t + \varepsilon_{g_{t+1}} \]
A.3. Solving

The system is given by 19 equations with 19 variables. The endogenous state variables \{ \hat{m}_i, \hat{b}_i, \hat{K}_i, \hat{m}^c_i, \hat{\epsilon}_i, \hat{\Lambda}_i \} include lambda and the nominal exchange rate in addition to the standard four variables, as Uhlig’s toolkit suggests that variables dated \( t-1 \) or earlier should be considered state variables (in the case of \( \hat{\epsilon}_i \)) while the matrix of other endogenous variables should be non-singular in order for its pseudo-inverse to exists, allowing to redeclare \( \hat{\Lambda}_i \) as other endogenous state variable instead. The other endogenous variables of the system are \{ \hat{\pi}_i, \hat{\delta}_i, \hat{\psi}_i, \hat{\beta}_i, \hat{\nu}_i, \hat{H}_i, \hat{Y}_i, \hat{I}_i, \Delta \hat{M}_t, \hat{\Gamma}_i \}, and the exogenous state variable are \{ \hat{\theta}_i, \hat{g}_i, \hat{z}_i \}.