Credit Frictions and Unexpected Credit Crunches

Sergio Salas †

Central Bank of Chile

Abstract

In this paper I develop a simple model of dynamic general equilibrium similar to the neoclassical growth model, but where credit flows are created by agents stochastically receiving investment opportunities that allow them to create new capital. Hence they may switch status over time from investors to workers and vice versa. Agents can issue equity up to a given fraction, so they can partially finance their investment costs externally and are in effect borrowing constrained. I characterize steady states and transitional dynamics in this environment and analyze the effects on allocations, asset prices and returns of a sudden, unexpected credit crunch: an exogenous, unanticipated decrease in the maximum fraction of investment costs that can be financed externally. I find that this type of unexpected shock generates a contraction in output and produces an heterogeneous response in the return on assets, depending on the agent's status.

Keywords: Credit frictions, credit crises, asset markets.

JEL Classification: E1, E44.

†E-mail: ssalas@bcentral.cl. This paper is a modified part of my doctoral dissertation, I am especially grateful for the advice and encouragement of my committee: Veronica Guerrieri, Kenneth Judd and Harald Uhlig.

1 Introduction

As the financial crisis began in late 2007, few people were aware that of how quickly it would escalate, with the freezing of many financial assets that had served as liquidity providers and the ensuing deep and prolonged recession. It seems that in order to understand the causes and features of the crises, we must look at models in which financial and liquidity factors play a central role. The role of liquidity and credit has been conceptualized in different ways in the literature; see for example Lucas [1980], Diamond and Dybvig [1983], Curdia and Woodford [2009], Wen [2009], Aiyagari [1994], Hugget [1993]. Some more recent papers that attempt to address different aspects of the effects of credit crunches and the role of financial intermediaries and non-traditional monetary policy are Guierrieri and Lorenzoni [2011], Kiyotaki and Moore [2008], Brunnermeier and Pedersen [2008], Adrian and Shin [2011] and Gertler and Kiyotaki [2011].

In this paper I build upon the work of Kiyotaki and Moore [2008], where liquidity needs are introduced by allowing some agents in the economy to become entrepreneurs in a broad sense, enabling them to transform goods into capital, to become creators of value or investors. The model is similar to the neoclassical model but agents are heterogeneous because in each period only some of them find investment projects that allow them to transform consumption goods into new capital. I call agents with investment projects "investors" and agents without them "workers". The probability of being an investor is the same for all individuals and the same each period; hence, their status may change each period but the economy will display the same aggregate fractions of investors and workers. They can trade claims on capital among themselves for the new units of capital created. Thus, the ownership structure of capital is heterogeneous and involves a portfolio where agents can issue claims on capital they manage and purchase claims on capital managed by others. Given that the status of an agent is a random variable, the model can be conceptualized as belonging to a class of models with uninsurable idiosyncratic risks, similar to the Bewely-Huggett-Ayagari class of models. In particular, investors can sell equity on the new units of capital created, which can be conceived of as borrowing, and there is an exogenous borrowing constraint, a maximum fraction of investment on which agents can issue claims on.

To simplify and obtain closed-form solutions for analytical tractability, I assume that capital depreciates completely in the production process and that agents have linear utility in consumption. I characterize steady states and dynamics. The steady state is an interesting situation to study, not only because it will serve as a point for approximation for the dynamics of the model but also because agents are perpetually changing their status in this heterogeneous framework and hence even in the steady state non-trivial trades take place among agents and the credit frictions plays a role in allocations and asset prices. I show that when the credit constraint is high enough, first-best, aggregate frictionless allocations are not reached and the economy displays lower output and consumption. In this situation, all investors saturate their credit constraint and issue the maximum possible equity to finance investment, but the constraint prevents a sufficient flow of resources among agents to support desirable amounts of capital creation.

I characterize the dynamics of the model and show that the equilibrium can be written as a two-variable system of difference equations in the price of equity and aggregate capital. I show the saddle-path stability property of the model and characterize the transition path towards the steady state of the variables in the economy. In this set-up I study an unexpected credit crunch – a permanent decrease in the maximum amount of equity investors can issue in the market – and show that this generates a contraction in output that gradually converges to a new lower level, an increase in consumption at impact followed by a decrease towards a new lower level and a heterogeneous response in return on assets. Depending on the agent's status when the shock hits and in subsequent periods, the return on assets may increase or decrease and converge to different steady state levels.

The rest of the document is organized as follows. Section 2 presents the model. Section 3 provides an analysis of the model and a characterization of the solution in the steady state. Section 4 characterizes the dynamic of the model and the credit crunch, while Section 5 concludes.

2 The Model

Environment

The economy is populated by a measure one of infinitely lived households or agents which are indexed by $i \in [0, 1]$. At time 0, each agent has the same amount of homogeneous capital. Each period they have a labor endowment normalized to unity, with no attached disutility.

Every period, each agent can receive with probability π and for free a technology that allows him to transform units of the consumption good into new capital; the occurrence of these opportunities are thus identical and independently distributed. This is the source of heterogeneity introduced by Kiyotaki and Moore [2008]. I refer to the agent who possesses the investment technology as an investor and the agent who does not possess the technology as a worker[1]. I define the status of the agent i as $z_{it} = \{1, 0\}$, to denote an investor and a worker, respectively. I also assume that capital depreciates completely in the production process; hence, individual capital accumulation evolves as:

$$k_{it+1} = \begin{cases} x_{it} & \text{for} \quad z_{it} = 1 \\ 0 & \text{for} \quad z_{it} = 0. \end{cases} \tag{1a}$$

At the beginning of each period each agent will supply the services of capital to a CRS technology

[1]Although investors also work, there is no associated utility loss.

firm whose optimization problem will be introduced later[2]. Investors can issue claims or equity on the capital they hold in any period. I define e_{it+1} as equity issued on managed new capital, where "manage" is defined as being taken to the firms that rent their services and s_{it+1} is the "self-claimed" part. Of course, for a worker both quantities are zero.

Let a_{it} be an agent's claims on capital managed by other agents prior to the production process, then the investor's net worth is $n_{it} = s_{it} + a_{it}$, while the worker's net worth is simply $n_{it} = a_{it}$.

Investors face the following credit constraint: θ is the maximum fraction of new units of capital he can issue claims or equity on, and therefore capital creation cannot be fully financed externally. Also, both investors and workers are borrowing-constrained because they cannot hold negative quantities of claims on capital managed by others. The restrictions can be written as:

$$0 \leq e_{it+1} \leq \theta x_{it}, \quad 0 \leq \theta \leq 1, \quad z_{it} = 1, \tag{1b}$$

$$a_{it+1} \geq 0, \quad z_{it} = \{0, 1\}. \tag{1c}$$

The budget constraints in units of the homogeneous good that agents face are given by:

$$c_{it} + x_{it} + q_t a_{it+1} \leq r_t(s_{it} + a_{it}) + w_t + q_t e_{it+1}, \quad z_{it} = 1, \tag{2a}$$

and

$$c_{it} + q_t a_{it+1} \leq r_t a_{it} + w_t, \quad z_{it} = 0. \tag{2b}$$

I have defined q_t as the price on claims traded, r_t is the rental rate on capital and w_t is the wage

[2]Hence, at the beginning of the period, capital is still productive; it is only after production that it depreciates completely.

rate.

The agent's optimization problem

Each agent has linear preferences over consumption without disutility of labor; hence, they maximize the discounted sum of consumption, conditional on a given individual state which is the status z. Over time, as their individual status evolves stochastically, they will differ in the amount of equity they hold, and the price of claims and the rental rate will also be relevant values for them.

Using the definition of net worth: $n_{it} = s_{it} + a_{it}$ and $n_{it} = a_{it}$ for investors and savers, respectively, it is possible to manipulate equations (1) and (2), to simplify agents' constraint set.

The budget constraint of an investor can be written using (1a) and (2a) as:

$$c_{it} + x_{it} + q_t n_{it+1} \leq r_t n_{it} + w_t + q_t x_{it}. \tag{3a}$$

Both of his liquidity constraints (1c) and (1b) can be combined into a single liquidity constraint:

$$n_{it+1} \geq (1 - \theta) x_{it}. \tag{3b}$$

The worker's budget constraint set can also be simplified by using (1a) and (2b) as:

$$c_{it} + q_t n_{it+1} \leq r_t n_{it} + w_t, \tag{3c}$$

with a single liquidity constraint by combining (1c) and (1c):

$$n_{it+1} \geq 0. \tag{3d}$$

In the restrictions above, we can interpret n_{it} as the demand for claims on capital by the individual i in time t, which is composed of self-claimed capital plus claims on capital managed by others. Note that even if there is a solution to this modified problem, it won't be possible to find a solution to the original problem; the composition of each agent's net worth will not be defined.

The firm's optimization problem

CRS firms rent capital and labor services each period and their problem is as follows:

$$\max_{K_t, L_t} \left[F(K_t, L_t) - r_t K_t - w_t L_t\right], \tag{4a}$$

With optimality conditions:

$$r_t = F_K(K_t^d, L_t^d),$$

$$w_t = F_L(K_t^d, L_t^d), \tag{4b}$$

The superscript denotes the demand for factors. Throughout this paper, I will use the Cobb-Douglas production function: $Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1$. A is normalized such that the steady state value of a frictionless level of capital of a representative agent economy equals one: $A = 1/(\alpha\beta)$.

2.1 Equilibrium

Given a sequence of prices of claims $\{q_t\}$ and prices of factors of production $\{r_t\}$, $\{w_t\}$, let $c_t(n, z)$, $n_{t+1}(n, z)$ and $x_t(n, z)$ be the policy functions for consumption, net worth and investment for an agent with net worth $n_{it} = n$ and status $z_{it} = z$. Let $\Psi(n, z)$ denote the joint distribution of net worth and current status in the population.

Definition *An equilibrium is a sequence of prices of claims and factors* $\{q_t, r_t, w_t\}$*, a sequence of policy functions* $\{c_t(n,z), n_{t+1}(n,z), x_t(n,z)\}$ *and a sequence of distributions* $\{\Psi_t(n,z)\}$ *such that, given an initial distribution* $\Psi_0(n,z)$*:*

1. $c_t(n,z)$*,* $n_{t+1}(n,z)$ *and* $x_t(n,z)$ *are optimal given* $\{q_t, r_t, w_t\}$*,*

2. *at given* $\{r_t, w_t\}$ *firms maximize profits,*

3. *The claims on capital market clears:*

$$\int k d\Psi(n,1) \equiv K_t^s = \int n d\Psi_1(n,1) + \int n d\Psi(n,0)$$

4. *the capital and labor markets clear:*

$$\begin{aligned} K^d &= K^s = K \\ L^d &= \int d\Psi(n,1) + \int d\Psi(n,0) = L^s = 1 = L \end{aligned}$$

5. *Investment demand equals savings:*

$$\sum_z \int n_{t+1}(n,z) d\Psi(n,z) = \int x(n,1) d\Psi(n,1)$$

3 Analysis

q_t is the key price that will induce agents to behave in a certain way; it is also, of course, the result of interactions among the whole population. To solve for the equilibrium, I assume that $q_t > 1$ and then find conditions such that close to the steady state this is indeed an equilibrium[3].

[3]As we will see, $q_t = 1$ will be indicative of an economy that attains first-best allocations. $q_t < 1$ cannot arise in equilibrium as no investor would be willing to incur the unit cost of creating capital when he can buy it in

By examining (3a) it is clear that if $q_t > 1$, investors have strong incentives to produce capital and sell as much equity as possible. Restriction (1b) however prevents them from doing this unboundedly, this restriction will be satisfied with equality. And, it is possible to combine both of the investor's constraints as:

$$c_{it} + \psi_t n_{it+1} \leq w_t + r_t n_{it}$$

$$n_{it+1} \geq 0, \quad c_{it} \geq 0, \quad z_{it} = 1. \tag{6a}$$

where I have defined the effective price that investors pay for purchasing net worth as: $\psi_t = (1 - \theta q_t)/(1-\theta)$[4]. As long as $q_t > 1$[5]:

$$0 < \psi_t < 1 < q_t. \tag{7}$$

Feasibility set (6) is isomorphic to the one that savers face (see equations (3c) and (3d)). But it is useful to keep in mind that investors are always supplying equity under this scenario because their liquidity constraint is binding[6].

Proposition 1. *The agent's policy functions for net worth and investment consistent with market equilibrium are given by:*

$$n_{t+1}(n,1) = \begin{cases} \frac{1}{\psi_t}\left(w_t + r_t n\right) & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] > \psi_t \\ \textit{indifferent} & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] = \psi_t \\ 0 & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] < \psi_t \end{cases} \tag{8a}$$

the market at a lower price.

[4] ψ_t can be thought of as the unit value of net worth that investors face. Since the investor can sell claims on one unit of investment up to θ, the effective cost is $1 - q_t\theta$, and to get one unit of net worth in this way he needs to create $1/(1-\theta)$ units of investment.

[5] In any equilibrium where $q_t > 1$ it is necessary also that $1 - q_t\theta > 0$, since as will be shown shortly, investors would face a negative effective price for capital accumulation, and hence their problem wouldn't have a solution.

[6] Investors are always liquidity constrained. The first inequality in (6a), which looks like the worker's liquidity constraint, is in fact the investor's non-negative constraint on investment.

$$x(n,1) = \begin{cases} \frac{1}{1-\theta q_t}\left(w_t + r_t n\right) & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] > \psi_t \\ \textit{indifferent} & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] = \psi_t \\ 0 & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] < \psi_t \end{cases} \tag{8b}$$

$$n'(n,0) = \begin{cases} \frac{1}{q_t}\left(w_t + r_t n\right) & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] > q_t \\ \textit{indifferent} & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] = q_t \\ 0 & \textit{if} \quad \beta E_t\left[\pi \frac{r_{t+1}}{\psi_{t+1}} q_{t+1} + (1-\pi) r_{t+1}\right] < q_t \end{cases} \tag{8c}$$

Proof. See the Appendix. □

Proposition 2. *The aggregate equilibrium of the economy is characterized by the following dynamic system:*

$$q_t = \beta E_t\left[\frac{r_{t+1}}{\psi_{t+1}} q_{t+1}\pi + r_{t+1}(1-\pi)\right] \tag{9a}$$

$$K_{t+1} = \frac{1}{1-\theta}\left(\frac{w_t}{\psi_t}\pi + \frac{r_t}{\psi_t}\pi K_t\right) \tag{9b}$$

$$r_t = \alpha A K_t^{\alpha-1}, \quad w_t = (1-\alpha) A K_t^{\alpha}. \tag{9c}$$

Proof. In the Appendix, I show that an equilibrium exists when equation (9a) is satisfied. In this case, the investors' opportunity cost of current consumption is high enough so that given the linearity of their preferences, they choose to consume nothing. Indeed, policy function (8a) states that all resources will be assigned to purchases of claims. The policy for workers (8c) implies that they are indifferent on how much to accumulate or consume.

By defining $N_{1t+1} \equiv \int n_{it+1} d\Psi(n,1)$, which aggregates net worth for investors under (9a), we have:

$$N_{1t+1} = \int \frac{w_t}{\psi_t} d\Psi(n,1) + \int \frac{r_t}{\psi_t} n_{it} d\Psi(n,1) = \frac{w_t}{\psi_t}\int d\varphi(n)\pi + \frac{r_t}{\psi_t}\int n_{it} d\varphi(n)\pi = \frac{w_t}{\psi_t}\pi + \frac{r_t}{\psi_t} N_t \pi$$

where φ is the marginal distribution of net worth. Note that it is possible to factorize the joint distribution as $\Psi(n, 1) = \varphi(n)\pi$ because of the i.i.d. assumption on shock status[7]. By market clearing for investment, we can find that the aggregate amount of net worth acquired by workers is $N_{0t+1} = \theta I_t = \theta K_{t+1}$. And hence by equilibrium in the market of claims we have: $K_{t+1} = N_{1t+1} + N_{0t+1}$ which gives equation (9b) and equations (9c) are result immediately from the optimality condition for firms plus market clearing in assets. □

3.1 Steady States

I study the the economy described in the previous section in the steady state. I provide conditions under which the economy attains a suboptimal equilibrium. The two key parameters will be given by π and θ. Note that there are still movements in this situation because the idiosyncratic shocks of status are perpetually buffeting the agents.

Proposition 3. For all values of θ, π such that

$$\pi + \alpha\beta\theta \geq \alpha\beta, \tag{10a}$$

the economy attains its first-best equilibrium where:

$$\beta r^* = 1, \quad I^* = K^* = q^* = 1, \quad Y^* = \frac{1}{\alpha\beta}, \quad C^* = \frac{1-\alpha\beta}{\alpha\beta}. \tag{10b}$$

Conversely, when

$$\pi + \alpha\beta\theta < \alpha\beta, \tag{10c}$$

[7]The timing of the model implies that when agents choose n_{it+1} they don't know the shock status z_{it} yet and hence both are independent.

the economy attains a suboptimal equilibrium where:

$$\beta\bar{r} < 1, \quad \bar{I} = \bar{K} < I^* = K^*, \quad \bar{q} > q^* = 1, \quad \bar{Y} < Y^*, \quad \bar{C} < C^*. \tag{10d}$$

Proof. See the Appendix. □

The economy needs to be sufficiently constrained in terms of having a small enough value of θ in order for the economy to attain a suboptimal equilibrium, as is stated in condition (10c). When the economy presents only mild credit constraints, enough resources flow among agents and the optimal aggregate values are obtained.

Note also that there are different measures of effective ex-post returns to capital accumulation depending on the agent's status. Four ex-post returns are identified, depending on the status of the agent. The returns can be defined as:

$$\bar{\mathcal{R}}_{11} = \frac{\bar{q}}{\bar{\psi}}\frac{\bar{r}}{\bar{\psi}}, \quad \bar{\mathcal{R}}_{10} = \frac{\bar{r}}{\bar{\psi}}, \quad \bar{\mathcal{R}}_{01} = \frac{\bar{q}}{\bar{\psi}}\frac{\bar{r}}{\bar{q}}, \quad \bar{\mathcal{R}}_{00} = \frac{\bar{r}}{\bar{q}} \tag{11}$$

which corresponds to an investor who remains as such next period, an investor who changes status to a worker, a worker who becomes an investor and a worker who remains a worker, respectively. Because $\bar{q} > 1 > \bar{\psi}$ when condition (10c) is satisfied, it is possible to establish a relationship between all the returns as:

$$\bar{\mathcal{R}}_{11} > \bar{\mathcal{R}}_{10} = \bar{\mathcal{R}}_{01} > \frac{1}{\beta} > \bar{r} > \bar{\mathcal{R}}_{00} \tag{12}$$

If condition (10a) is satisfied then the above inequalities will be replaced with equalities; that is, all returns will be equal to the discount rate $1/\beta$ as in the standard neoclassical growth model.

4 Dynamics

The system can be written as:

$$q_t = \pi\beta\frac{(1-\theta)q_{t+1}}{1-\theta q_{t+1}}\alpha A K_{t+1}^{\alpha-1} + (1-\pi)\beta\alpha A K_{t+1}^{\alpha-1} \tag{13a}$$

$$K_{t+1} = \pi A\frac{K_t^{\alpha}}{1-\theta q_t} \tag{13b}$$

This is a nonlinear system of equations, I characterize the properties of the solution by linearizing it near the steady state:

$$\begin{bmatrix} q_{t+1}-\bar{q} \\ K_{t+1}-\bar{K} \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha\beta}\frac{1-\alpha\theta\bar{q}}{1-\theta} & \frac{1-\alpha}{\beta}\frac{\bar{\psi}\bar{q}}{\bar{K}} \\ \frac{\theta}{1-\theta}\frac{\bar{K}}{\bar{\psi}} & \alpha \end{bmatrix} \begin{bmatrix} q_t-\bar{q} \\ K_t-\bar{K} \end{bmatrix} \tag{14}$$

Proposition 4. The system (14) displays the saddle-path stability property.

Proof. The characteristic equation for the matrix associated with the system can be written as:

$$h(\lambda) \equiv \lambda^2 - \left[\alpha + \frac{1-\alpha\theta\bar{q}}{\alpha\beta(1-\theta)}\right]\lambda + \frac{1-\theta\bar{q}}{\beta(1-\theta)} = 0$$

Then we have:

$$h(-\infty) = +\infty, h(\infty) = +\infty, h(0) = \frac{1-\theta\bar{q}}{\beta(1-\theta)} > 0, h(1) = -(1-\alpha)\left[\frac{1-\alpha\beta(1-\theta)}{\alpha\beta(1-\theta)}\right] < 0$$

which indicates that one root is less than one and the other is greater than one. □

The saddle-path stability property of the model implies that for any initial value of the stock of capital close to the steady state, the price of claims q will adjust to put the system on the stable path.

4.1 Trajectory to the steady state

Having established the saddle-path stability of the model, I implement the parametric path method of Judd [2002] to numerically solve the model in the transition to the steady state. The reader is referred to his paper for further details[8]. In this exercise I assume that the stock of capital is below the steady state and solve the model for the transition. The price of capital will have to endogenously adjust in such a way as to put the system in the saddle path of the model[9]. Figure 1 shows the value of θ which is perceived correctly by agents as being constant.

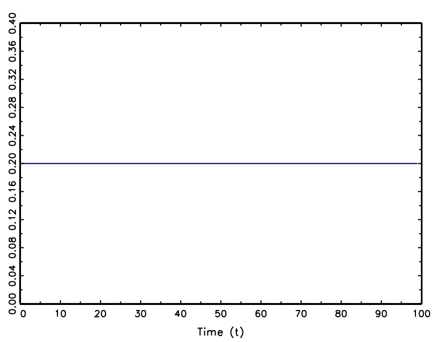

Figure 1: θ

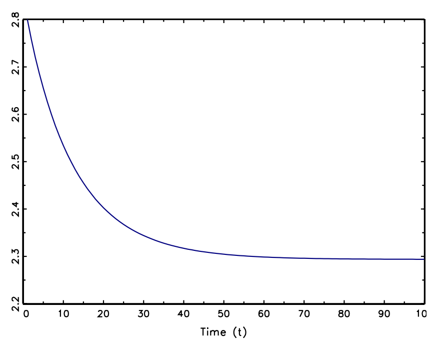

Figure 2: q_t

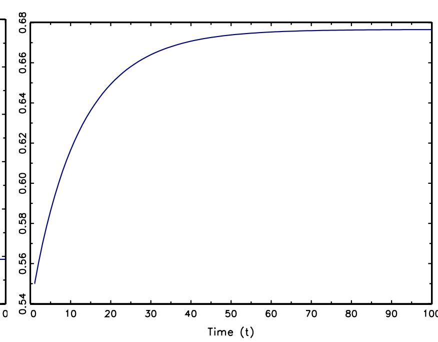

Figure 3: ψ_t

The initial value of the stock of capital is lower than its steady state counterpart. The price of capital is converging to the steady state from above, as figure 2 shows. The effective price of capital converges to the steady state from below. Observing figures 4 to 6, it is evident that the properties of the model are similar to the neoclassical growth model, with a gradual increment in the stock of capital, output and consumption towards the steady state. Figures 7 and 8 show the time path for factor payments. Several measures of ex-post returns are shown in figure 9;

[8] One advantage of the Judd [2002] method for solving the model is that the solution is approximated with polynomials and no shooting algorithm is needed. Also, unlike methods such as Fair and Taylor, no truncation is incorporated into the solution.

[9] The values used for this simulation are: $\alpha = 0.36, \beta = 0.95, \theta = 0.2$ and $\pi = 0.1$.

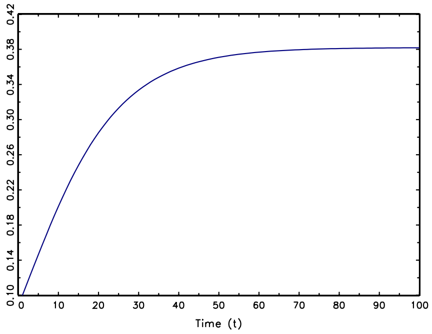

Figure 4: K_t

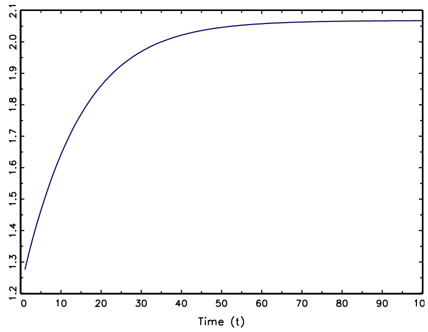

Figure 5: Y_t

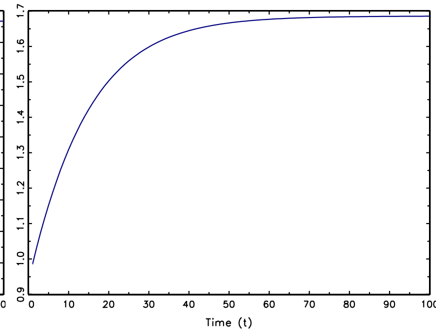

Figure 6: C_t

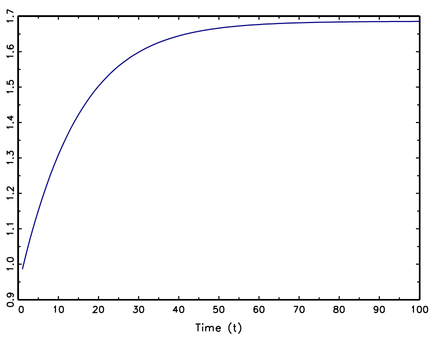

Figure 7: r_t

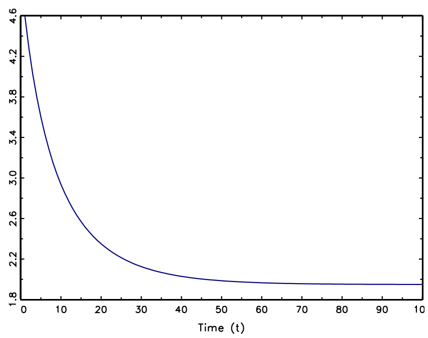

Figure 8: w_t

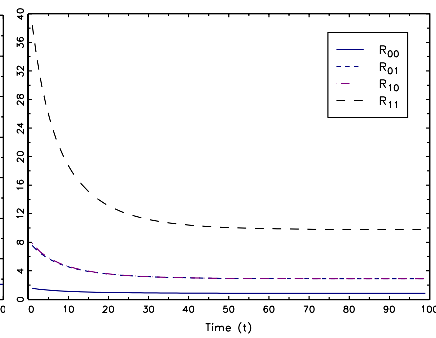

Figure 9: $\mathcal{R}_{00}, \mathcal{R}_{01}, \mathcal{R}_{10}, \mathcal{R}_{11}$

these are analogous to (11) in the dynamic context and hence are defined as:

$$\mathcal{R}_{11} = \frac{q_{t+1}}{\psi_{t+1}}\frac{r_{t+1}}{\psi_t}, \quad \mathcal{R}_{10} = \frac{r_{t+1}}{\psi_t}, \quad \mathcal{R}_{01} = \frac{q_{t+1}}{\psi_{t+1}}\frac{r_{t+1}}{q_t}, \quad \mathcal{R}_{00} = \frac{r_{t+1}}{q_t} \tag{15}$$

Note that an investor who maintains his status receives the highest returns and a worker who maintains his status receives the lowest returns, and these relationships do not change as steady state is approached[10].

[10] We know from the steady state analysis that a worker who becomes an investor and an investor who becomes a worker receive the same return in the steady state, but not during the transition, although for this parametrization they are roughly equal.

4.2 An unexpected credit crisis

In this section I explore the consequences for the model economy of a sudden, unexpected decrease in the parameter θ. There is an abrupt drying-up of the market and investors are unable to sell as much as equity as before. Note that this is not the same as a direct shock to investment, but a tightening constraint on selling claims to finance investment creation. I assume that the economy is at a steady state in period 0 where $\theta = 0.2$ and at the beginning of period 1 there is a sudden and unexpected permanent increase in the borrowing constraint with $\theta = 0.1$. Figure 10 shows the value of θ which suddenly drops by half. The price of capital increases at

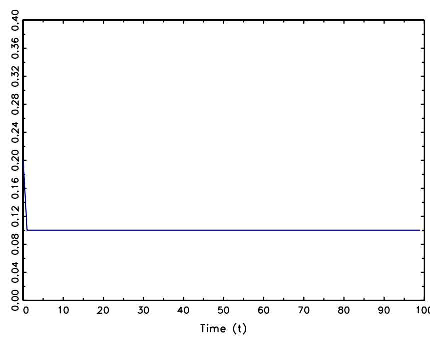

Figure 10: θ

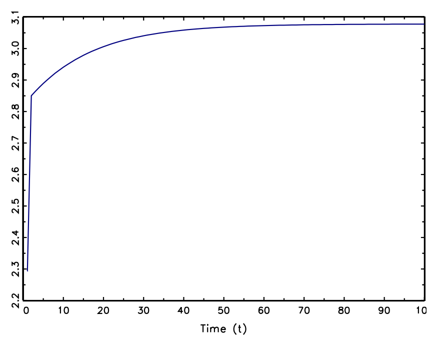

Figure 11: q_t

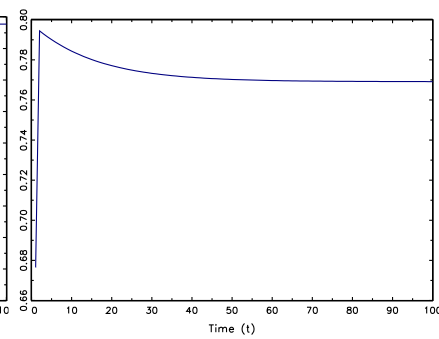

Figure 12: ψ_t

impact and converges gradually to the new steady state, as shown in figure 11. The effective price of capital increases and overshoots its steady state value, as figure 12 shows. Figures 13

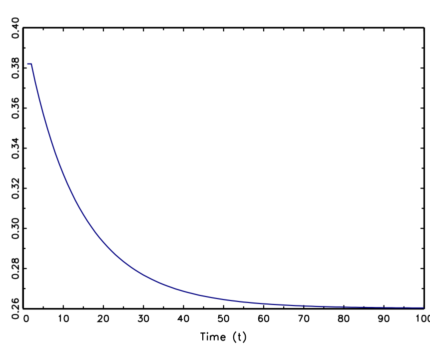

Figure 13: K_t

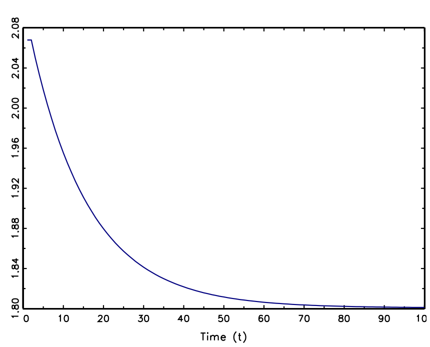

Figure 14: Y_t

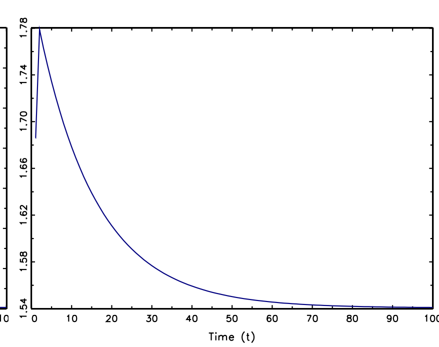

Figure 15: C_t

and 14 show the contractive effects of the shock. There is a permanent contraction in economic production; figures 16 and 17 show the expected paths of convergence of a decreasing level of capital towards steady state. Figure 15 shows that consumption increases upon impact, since investors suddenly deleverage because they cannot finance more capital creation. The amount of goods that workers were initially selling to investors are consumed; there is no point in saving more, as the return on savings is correctly perceived to be low for the worker. Evidence of this is in figure 18, where $\mathcal{R}_{00}$ is the same at impact and expected to decrease later while $\mathcal{R}_{01}$ increases upon impact but is later expected to decrease as well. Figure 18 shows the ex-post effective

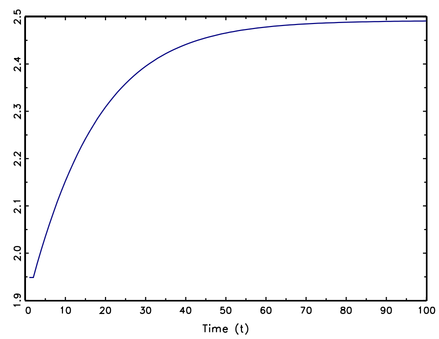

Figure 16: r_t

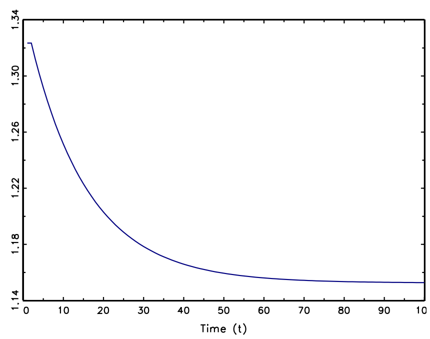

Figure 17: w_t

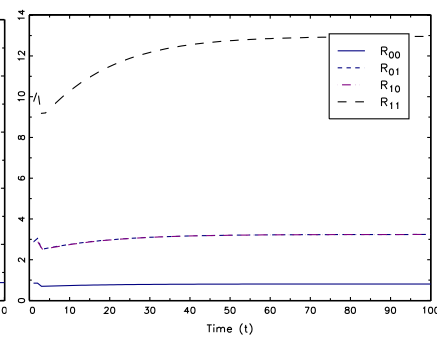

Figure 18: $\mathcal{R}_{00}, \mathcal{R}_{01}, \mathcal{R}_{10}, \mathcal{R}_{11}$

return to investors; it should be noted that upon impact the return is increased, then declines for a period and then gradually increases. Hence, the unexpected change may be beneficial in terms of returns for some individuals in this economy, depending on their status. It is impossible to determine the individual consumption trajectories for consumption and utility in the economy; even though investors do not consume, they eventually have worker status during which time they consume small or large amounts, depending on the time elapsed without a status change. We know that in the aggregate consumption monotonically decreases towards the steady state after the initial jump. If we consider that the price of claims has a counterpart in reality, perhaps the closest would be the stock market, which falls during crises rather than increasing as in the model. In reality, however, there are other assets that agents resort to during periods of stress such as commodities, other currency, government bonds and others, in a so-called "flight to quality." In the model for tractability I have precluded of this possibility because there is a single asset.

5 Conclusions

In this paper I have developed a model in which credit flows arise because agents differ in terms of access to investment opportunities; agents who do not have investment opportunities partially finance the credit needs of agents who have investment opportunities. The model developed is similar to the decentralized neoclassical growth model because agents own productive factors and rent their services to constant returns to scale production firms, but because they are heterogeneous, they engage in asset transactions.

Credit constraints in financing investment are exogenously imposed and hence represent short-cuts to a more complex modeling of informational or agency problems in the financial markets.

Since only some agents enjoy the benefits of new capital creation, the model can be thought of as having two frictions, one real and the other financial. The former is the probability of finding investment projects and the latter the credit constraint. I characterize the steady state and show that when both frictions are significant enough, aggregate outcomes are suboptimal compared to the frictionless case, but there is no need for frictions to be completely absent to reach aggregate efficiency.

Finally, I show the dynamics of the economy under frictions and the effects of an unexpected credit crunch that prevents investors from issuing as much equity as before. I show that this produces a gradual and permanent contraction in production. The status of an agent in the transition following the shock determines their return on capital accumulation, so some agents may gain and others may lose. Hence this model reveals that financial crises may be beneficial to certain individuals in the economy. Aggregate consumption, however, displays suboptimal behavior because except for the period of the shock, aggregate consumption decreases monotonically towards a new lower steady state level.

Appendix

Proof of Proposition 1

Proof. I use the guess and verify method to solve the model[11]. At the individual level there are two states: the status of the agent and net worth. There are also economy-wide states which are q and r. The guess is that the value function is linear in the state n:

$$v(n, z) \quad = \quad A_z(q, r) + B_z(q, r)n, \quad z = \{1, 0\}$$

where A_z and B_z are coefficients to be determined that depend on the price of claims and the rental rate on capital. If these value functions exist, we can write the Bellman equation for the investor as:

$$v(n, 1) = \max_{n'} E\left[w + rn - \psi n' + \beta \bar{A}(q', r') + \beta \bar{B}(q', r')n'\right]$$

such that:

$$n' \geq 0 \quad c \geq 0$$

The value function for the saver is given by:

$$v(n, 0) = \max_{n'} E\left[w + r - qn' + \beta \bar{A}(q', r') + \beta \bar{B}(q', r')n'\right]$$

such that:

$$n' \geq 0 \quad c \geq 0$$

where $\bar{B}(q, r) = \pi B_1(q, r) + (1 - \pi)B_0(q, r), A = \pi A_1(q, r) + (1 - \pi)A_0(q, r)$.

Given the linearity of the value functions, computing the policies is straightforward. For the investor, we have:

[11]I use recursive dynamic programming and omit time subscripts in this appendix.

$$n'(n,1) = \begin{cases} \frac{1}{\psi}(w+rn) & \text{if} \quad \beta E\bar{B}(q',r') > \psi \\ \text{indifferent} & \text{if} \quad \beta E\bar{B}(q',r') = \psi \\ 0 & \text{if} \quad \beta E\bar{B}(q',r') < \psi \end{cases}, \quad c(n,1) = \begin{cases} 0 & \text{if} \quad \beta E\bar{B}(q',r') > \psi \\ \text{indifferent} & \text{if} \quad \beta E\bar{B}(q',r') = \psi \\ w+rn & \text{if} \quad \beta E\bar{B}(q',r') < \psi \end{cases}$$

and:

$$x(n,1) = \begin{cases} \frac{1}{1-q\theta^s}(w+rn) & \text{if} \quad \beta E\bar{B}(q',r') > \psi \\ \text{indifferent} & \text{if} \quad \beta E\bar{B}(q',r') = \psi \\ 0 & \text{if} \quad \beta E\bar{B}(q',r') < \psi \end{cases}$$

Similarly. for the saver we have:

$$n'(n,0) = \begin{cases} \frac{1}{q}(w+rn) & \text{if} \quad \beta E\bar{B}(q',r') > q \\ \text{indifferent} & \text{if} \quad \beta E\bar{B}(q',r') = q \\ 0 & \text{if} \quad \beta E\bar{B}(q',r') < q \end{cases}, \quad c(n,0) = \begin{cases} 0 & \text{if} \quad \beta E\bar{B}(q',r') > q \\ \text{indifferent} & \text{if} \quad \beta E\bar{B}(q',r') = q \\ w+rn & \text{if} \quad \beta E\bar{B}(q',r') < q \end{cases}$$

In principle, there are several possibilities that might arise in a partial equilibrium set-up. For example, in terms of prices, it might be that the gain of accumulating one unit of capital, $\beta E\bar{B}(q',r')$ is higher than the cost, q and ψ for savers and investors, respectively. This would imply that agents consume as much as possible, and therefore savers will be liquidity constrained and investors would not create any new units of capital. This situation is clearly not an equilibrium as there cannot be a stationary equilibrium with positive consumption. In the reverse case, where the benefit of accumulation is higher than the cost for both investors and savers: $\beta E\bar{B}(q',r') > q > \psi$, no agent will ever consume, and this clearly is not an equilibrium.

Since the slope of the constraint for the saver is smaller than that of the investor, it is natural to guess that general equilibrium will display the investor accumulating as much as possible and the saver either indifferent or at a corner with the binding liquidity constraint. A look ahead to the equilibrium of the model, however, rules out the latter case for the investor, as there would not be any demand for the equity the investor is issuing. Hence, if a solution exists it must be under the case:

$$\psi < \beta E\bar{B}(q', r') = q$$

Finding the undetermined coefficients

To find the undetermined coefficients of the value functions, I consider[12]:

$$\begin{aligned} n'(n,1) &= \frac{1}{\psi}(w + rn), \quad c(n,1) = 0 \\ n'(n,0) &= \frac{1}{q}(w + rn), \quad c(n,0) = 0 \end{aligned}$$

With these policies I can write the Bellman equations as:

$$\begin{aligned} A_1(q,r) + B_1(q,r)n &= \beta E\bar{A}(q',r') + \frac{1}{\psi}(w+rn)\beta E\bar{B}(q',r') = \beta\bar{A} + \frac{q}{\psi}(w+rn) \\ A_0(q,r) + B_0(q,r)n &= \beta E\bar{A}(q',r') + \frac{1}{q}(w+rn)\beta E\bar{B}(q',r') = \beta\bar{A} + w + rn \end{aligned}$$

where I have used also: $\beta\bar{B}(q', r') = q$. Hence I find the values for B_1 and B_0[13]:

$$\begin{aligned} B_1(q,r) &= \frac{q}{\psi}r \\ B_0(q,r) &= r \end{aligned}$$

We have that $B_1 > B_0$ if $q > 1$ and $B_1 = B_0$ only if $q = 1$. $\square$

Proof of Proposition 3

[12]Note that given that the worker is indifferent between any combination of consumption and net worth, the case where he consumes zero and accumulates as much as possible will give the same value function as any combination of both variables.

[13]It is not possible to find the values of A_0 and A_1, but they are unimportant for characterizing an agent's decisions.

Proof. System 2 at steady state can be simplified to two equations involving K and q only:

$$\bar{q} = \pi\beta\frac{(1-\theta)\bar{q}}{1-\theta\bar{q}}\alpha A\bar{K}^{\alpha-1} + (1-\pi)\beta\alpha A\bar{K}^{\alpha-1} \tag{16}$$

$$\bar{K} = \pi A\frac{\bar{K}^{\alpha}}{1-\theta\bar{q}} \tag{17}$$

The solution for this system delivers:

$$\bar{q} = \frac{\alpha\beta(1-\pi)}{\pi(1-\alpha\beta)+\alpha\beta\theta}, \quad \bar{K} = \left(\frac{\pi(1-\alpha\beta)+\alpha\beta\theta}{\alpha\beta[1-\alpha\beta(1-\theta)]}\right)^{\frac{1}{1-\alpha}} \tag{18}$$

Placing equation (10a) at equality into these values we obtain $\bar{q} = 1$ and $\bar{K} = 1$ and by direct substitution into the relevant equations, we obtain the result in (10b). By direct computation, setting $\bar{q} > 1$ or $\bar{K} < 1$, we obtain condition (10c); in this case the value of capital is smaller than the first-best and so are the output and consumption that yield the relationships in (10c). $\square$

References

Tobias Adrian and Hyung Shin. "Financial Intermediaries and Monetary Economics". *Handbook of Monetary Economics*, Chapter 12:601–650, 2011.

Rao Aiyagari. "Uninsured Idiosyncratic Risk and Aggregate Saving". *The Quarterly Journal of Economics*, 109(3):659–684, 1994.

Marcus Brunnermeier and Lasse Pedersen. "Market Liqudity and Funding Liquidity". *The Review of Financial Studies*, 00(02008):1–38, 2008.

Vasco Curdia and Michael Woodford. "Credit Frictions and Optimal Monetary Policy". Working paper, 2009.

Douglas Diamond and Philip Dybvig. "Bank Runs, Deposit Insurance, and Liquidity". *Journal of Political Economy*, 91:401–419, 1983.

Mark Gertler and Nobuhiro Kiyotaki. "Financial Intermedaition and Credit Policy in Business Cycle Analysis". *Handbook of Monetary Economics*, Chapter 11:547–599, 2011.

Veronica Guierrieri and Guido Lorenzoni. "Credit Crises, Precautionary Savings and the Liquidity Trap". Working paper, University of Chicago and MIT, 2011.

Mark Hugget. "Precautionary Saving and Social Insurance". *Journal of Political Economy*, 103: 360–399, 1993.

Kenneth Judd. "The parametric path method: an alternative to Fair-Taylor and L-B-J for solving perfect foresight models". *Journal of Economic Dynamics and Control*, 26, 2002.

Nobuhiro Kiyotaki and John Moore. "Liquidity, Business Cycles, and Monetary Policy". Working paper, Priceton University, 2008.

Robert Lucas. "Equilibrium in a Pure Currency Economy". *Economic Inquiry*, 18(2):203–220, 1980.

Yi Wen. "When Does Heterogeneity Matter?". Working paper, Federal Reserve Bank of St. Louis, 2009.