A Model of Endogenous Growth With Moral Hazard: A Case For Microfinance
(Outline)

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Abstract

We present an overlapping-generation model of endogenous growth, where individuals invest in education as young, use the achieved human capital together with physical capital as middle aged, and consume as old. It is wellknown that in models of this type it is important for achievement of efficiency that either credit institutions or welfare policies are availabe to finance education and possibly retirement. In our present case, we assume that financial intermediation in providing credits for education is faced with moral hazard on the side of borrowers who may choose more or less risky educational investments. We show that using community liability for educational credits may increase the access to education and thereby improve welfare in the economy.

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1. Introduction

In recent theoretical models of growth, human capital plays a fundamental role, cf. e.g. Lucas (1988), Romer (1990). Investing in human capital is a crucial prerequisite of economic development, and the way in which society provides for such investment is correspondingly important. One of the main features of this investment process is the allocation of suitable

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education to individuals. Education is costly and will in most countries be provided for by
the society to a limited degree; individuals will have to put up the means for their education,
quite unfortunate since this investment is needed at a point in the individual’s lifetime when
only few resources are available for this purpose.

Theoretical investigation of this problem, taking into account the life cycle of individuals
and the timing of individual investment, has shown that with lack of capital markets for fi-
nancing investment in human capital, the allocation in society is inefficient, both statically
and dynamically, cf. Boldrin and Montes (2005), and this inefficiency calles for a system of
subsidies for education together with pension for the elderly, the latter phenomenon being
connected with the first one. However, this will work well only in the context of full in-
formation with respect to the individual choice of investment in human capital. In the case
where several investment strategies are possible, some of them being inferior from society’s
point of view but advantageous for the individual, there is a problem of moral hazard in-
volved, which may restrict the amount of investment in human capital and thereby society’s
welfare.

One way out of this dilemma is microfinance, here interpreted as in Stiglitz (1990),
where education is financed by loans given under group liability. This liability will improve
the payoff of the loans while putting an additional obligation on the individuals. However,
it has the important additional effect that banks can grant larger credits without forcing
borrowers into moral hazard, so that the overall investment in human capital increases with
the consequential results on growth and welfare.

We follow the approach of Boldrin and Montes (2005), Docquier, Paddison and Pestieau
(2007) and consider an overlapping generation model with three overlapping generations,
so that each generation lives for three consecutive periods, a first one where investment in
education is carried out, a second period with (instantaneous) production, where input is
human and physical capital and output is the consumption good, which can be used imme-
diately for consumption or transformed to capital goods. In the last period, the individual
only consumes, and this third period is included to trace the connection between financing
education and pensions.

2. The model

We consider an overlapping generations model, where individuals individuals live for three
consecutive periods. There is a single consumption good, and all individuals have the same
intertemporal utility function $u(c_m, c_o)$, where $c_m$ and $c_o$ denote consumption in the second
and third period of life; they may use the consumption good in the first period of life, namely
as input for production of human capital, but they do not derive utility from this use. The
generations are all of a fixed size, which is taken to be large, so that at a larger stage we can assume the law of large number to be at work. Technically, the set of consumers is identified with the interval \([0, 1]\).

An individual of the generation \(t\) (which is the generation born at \(t - 1\) and producing the consumption good in period \(t\)) can choose an investment using an education technology of type \(j\) with an investment function given by

\[ h_t = \nu_j H_j(e_{t-1}, h_{t-1}), \tag{1} \]

where \(h_t\) is the output of human capital obtained in period \(t\), \(e_{t-1}\) is the input of the consumption good at \(t - 1\), and \(h_{t-1}\) is human capital available at \(t - 1\). The investment function \(H_j\), assumed to be homogeneous of degree 1, is subject to a multiplicative random shock, taking the values 1 or 0 with probabilities \(\pi_j\) and \(1 - \pi_j\), respectively. The shocks are independent between individuals, so that an individual choosing an investment of type \(j\) will produce the human capital

\[ h_t = H_j(e_{t-1}, h_{t-1}) \]

with probability \(\pi_j\) and 0 with otherwise. The choice of type \(j\) is private information at time \(t - 1\), whereas the payoff according to (1) is assumed to be observable.

The index \(j\) on the investment function describes the technology, and it is assumed that there is more than one technology from which the individual may choose. Following the tradition in the literature, we shall assume that the type \(j\) can be either \(G\) or \(B\), where

\[ \pi^*_G H_G(e_{t-1}, h_{t-1}) \geq \pi^*_B H_B(e_{t-1}, h_{t-1}) \]

for all \(i\) and \((e_{t-1}, h_{t-1})\). For reasons to become clear as we proceed, the choice of technology will be the same for all individuals in the generation.

In the second period, the human capital obtained is used for the production of the consumption good. We assume that production is carried out in single firm with aggregate technology given by

\[ y_t = F_t(k_t, h_t), \]

where \(y_t\) is total production of the consumption good at time \(t\), \(k_t\) is the input of (physical) capital, and \(h_t\) is the input of human capital. Physical capital at \(t\), \(k_t\) belongs to the the generation \(t - 1\), who sells it to the firm and uses the proceeds for consumption.

Individuals are born with no endowments (except the human capital available), and in order to carry out investment in education, they must borrow the amount \(e_{t-1}\) from the previous generation \(t\) and pay back in the next period. Since the payoff of the investment is uncertain, the lenders, who are the individuals of the previous generation, will have to decide on the
basis of expected outcomes of the loan contracts.

For the decisions of the economic agents in our model, expectations at time \( t \) about future prices play a crucial role, and in accordance with the overlapping generations literature, we shall consider equilibria where these expectations are actually confirmed in the equilibrium allocation. Since there is randomness in our model, the equilibrium should be described as a stochastic process, but we shall assume that there are many individuals so that the law of large numbers apply, allowing us to work with averages instead.

Now we consider the choices of an individual in generation \( t \); at \( t - 1 \), this individual must choose level of investment in education (and indebtedness) \( e_{t-1} \) as well as technology \( j \). Then, at \( t \) the individual must repay the loan from last period and and buy consumption \( c_t \) as well as save for \( t + 1 \) purchasing physical capital. The choices are subject to the budget constraints

\[
\begin{align*}
e_{t-1} & \leq \bar{e}_{t-1} \\
p_t(c_t + s_t) + m_t & \leq \max \{w_th_t - R_te_{t-1}, 0\}, \\
p_{t+1}d_{t+1} & \leq p_{t+1}s_t + r_{t+1}m_t.
\end{align*}
\]

Here the first inequality, relating to time \( t - 1 \), expresses the quantitative restriction on loans for education, to be discussed below, the next inequality states that at time \( t \), the value of consumption and savings, both in the form of purchase of capital goods or as monetary deposit in the bank, cannot exceed the value of the human capital sold to the firm, with deduction of the debt; we allow for bankruptcy, in which case the net income at \( t \) is 0. Finally, the third inequality says that value of consumption in the third year of life is limited by the value of savings, both in the form of capital goods to be sold to the firm, and monetary savings. Here \( r_{t+1} \) is the deposit rate, the repayment at \( t + 1 \) per unit of money deposited at time \( t \).

3. Financial intermediation under conditions of moral hazard

Now we turn to the bank, which at date \( t \) chooses a credit limit \( \bar{e}_t \) for the new-born borrowers as well as a payoff rate \( R_{t+1} \) for the loans. For simplicity, we assume that the bank is a non-profit firm, and its objective is to balance the incomes from lending with the funding cost, or, alternatively, that competition and free entry forces profits down to zero. Given that investors choose technology \( j \), the operations of the bank must satisfy the solvency condition

\[r_{t+1}m_t \leq \pi_jR_{t+1}e_t,\]
where \( j \) specifies the technology chosen by investors, saying that its obligations towards depositors at date \( t + 1 \) must be satisfied through the repayments of the loans (note that only the fraction \( \pi_j \) of the borrowers will succeed).

Since the choice of technology is unobservable to the bank, it must conduct is business in such a way that the loss-inducing choice of \( B \) is avoided. The repayment rate \( R_t \) cannot do this if left alone: If the investor should choose \( G \), then it must be the case that

\[
\pi_G u^\circ(H_G(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_t) = \pi_B u^\circ(H_B(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_t),
\]

where \( u^\circ(w, r_t) \) (the indirect utility function given the budget and the rate of savings) is defined as the maximum of \( u(c_t, d_t) \) over all \((c_t, d_t)\) which satisfy

\[
\begin{align*}
c_t + s_t & \leq w, \\
d_t & \leq r_{t+1} m_t;
\end{align*}
\]

here the first inequality states that the consumption and savings (in the form of deposits in the bank) at date \( t \) should not surpass the amount of the good available at that date, and the second inequality is the budget constraint for the next period.

Assuming differentiability of \( u^\circ(w, r_t) \) and writing

\[
(u^\circ)'_j = \frac{\partial u^\circ(H_j(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_t)}{\partial w}, \quad j = G, B,
\]

we have that

\[
\begin{align*}
\frac{\partial}{\partial R_t} [\pi_G u^\circ(H_G(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_t) - \pi_B u^\circ(H_B(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_t)] \\
= (\pi_B(u^\circ)'_B - \pi_G(u^\circ)'_G) e_{t-1} & \neq 0
\end{align*}
\]

for \( e_{t-1} > 0 \), so that we can apply the implicit function theorem to get that the locus of the pairs \((e_{t-1}, R_t)\) for which \( B \) and \( G \) are equally good defines \( R_t \) as a differentiable function \( \rho \) of \( e_{t-1} \) with

\[
\frac{d\rho}{de_{t-1}} = -\frac{\pi_G(u^\circ)'_G(H_G' - R_t) - \pi_B(u^\circ)'_B(H_B' - R_t)}{(\pi_B u^\circ'_B - \pi_G u^\circ'_G)e_{t-1}}, \quad (7)
\]

where for simplicity we have written \( H_j' \) for \( \partial H_j(e_{t-1}, h_{t-1})/\partial e_{t-1}, \quad j = G, B. \) Using that \((u^\circ)'_G > (u^\circ)'_B\) by decreasing marginal utility, we have that the denominator is negative, and assuming that \( \pi_B(u^\circ)'_B H_B' > \pi_G(u^\circ)'_G H_G' \) for all \((e_{t-1}, R_t)\) such that (6) holds, we get that also the numerator is negative, so that \( \frac{d\rho}{de_{t-1}} > 0. \)

Following Stiglitz (1990), we assume that there is some fixed cost connected with edu-
cation, in the sense that there are constants $0 \leq e_G < e_B > 0$ such that

$$H_j(e_{i-1}) = 0 \text{ for } e_{i-1} < e_j.$$ 

Given that the quantity in (7) is negative, we conclude that technology $G$ is chosen if $(e_{i-1}^t, R_t)$ is in the area to the left of the curve defined by (6), denoted by $G$, whereas $B$ is chosen in the area $B$ to the right of this curve.

In order to fund the loans offered to the newborn investors at $t-1$, the financial intermediary must receive deposits from the middle-aged and pay them deposit rate $r_t$. The financial intermediary can ask a loan rate $R_t$ and set an upper bound $\bar{e}_{t-1}$ on the amount to be borrowed by any investor. Assuming that $H_j^\prime$ is big enough the investor will prefer a higher level of $e_{i-1}$ to a lower one, so the choices of the individuals will always be at the upper limit $\bar{e}_{t-1}$ defined by

$$\bar{e}_{t-1} = \rho^{-1}(R_t).$$

We can now rewrite the funding condition (5) with equality as

$$r_t m_{t-1} = \pi_G R_t \rho^{-1}(R_t). \tag{8}$$

### 4. Equilibrium in the decentralized economy with moral hazard

Now we may formulate the conditions for an equilibrium in the model. Each individual in generation $t$ chooses amount of education $e_{i-1}$ as young, consumption $c_{i}$ and deposit $m_{i}$ as middle aged, and consumption $d_{i}$ as old. Financial intermediaries choose the loan and deposit rates $R_{t}$ and $r_{t}$ as well as the credit limit $\bar{e}_{t-1}$. Producers choose the input of physical and human capital so as to maximize profits $w$.

Formally, an equilibrium is a collection of choices $((e_{t-1}, c_{t}, s_{t}, m_{t}, d_{t})_{i \in I}$ by individuals, $(R_{t}, r_{t})$ by financial intermediaries, and $(y_{t}, k_{t}, h_{t}))_{t = 0}^{\infty}$ by firms, such that at each $t$

(i) for all individuals $e_{t-1} = \rho^{-1}(R_{t})$ and $(c_{t}, m_{t}, d_{t})$ maximizes $u(c_{t}, d_{t})$ subject to (3)-(4) with $h_{t} = H_G(e_{t-1}, h_{t-1})$

(ii) $(R_{t}, r_{t})$ satisfies (8)

(iii) $(y_{t}, k_{t}, H_G(e_{t-1}, h_{t-1}))$ maximizes $y_{t} - k_{t}$ subject to the constraint $y_{t} \leq F(k_{t}, h_{t})$, (iv) commodity and factor markets balance: $e_{t+1} + (c_{t} + m_{t}) + d_{t-1}) \leq y_{t}, k_{t} = m_{t-1}$.

The conditions are standard – individuals choose so as to maximize utility, banks satisfy the funding condition developed above, producers maximize profits, and there is overall balance in the markets for consumption and physical capital.
5. Equilibrium under joint liability

Following the ideas of Stiglitz (1990) for formalizing microfinance, we shall now introduce the possibility of group liability, so that borrowers have to pay back not only their own, but also some part \( q < 1 \) of the defaulted borrowers’ loans. For simplicity, we assume that groups consist of borrowers of the same type (type may be observable to borrowers even if not to lenders), and the presence of group liability has the effect of increasing the probability that the bank gets the borrowed money back. In the case that the group consists of exactly two borrowers, the expected payoff to the bank becomes

\[
\pi_G R_t e_{t-1} + 2\pi_G (1 - \pi_G) q R_t e_{t-1} = \pi_G (3 - 2\pi_G) R_t e_{t-1}.
\]

Now the solvency constraint has become less binding, and this will allow the financial intermediary to reduce the loan rate \( R_t \), compensating for the increased burden put on the borrowers by the joint liability.

This reduction in rates, in its turn, has an effect on borrowers’ choices of technique. The expected utility of the borrower of type \( i \) choosing technique \( j \) is

\[
(\pi_j)^2 u^o_j (H_j(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_{t+1}) + \pi_j (1 - \pi_j) v(\pi_j (H_j(e_{t-1}, h_{t-1}) - (1 + q) R_t e_{t-1}, r_{t+1}).
\]

Differentiating w.r.t. \( q \) at \( q = 0 \), we get the expression

\[
\pi_j (1 - \pi_j) (u^o_j)' (H_j(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_{t+1}) [-R_t e_{t-1}].
\]

From \( u^o_B (H_G(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_{t+1}) > u^o_G (H_G(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_{t+1}) \) we get that

\[
(u^o_B)' (H_B(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_{t+1}) < (u^o_G)' (H_G(e_{t-1}, h_{t-1}) - R_t e_{t-1}, r_{t+1}),
\]

and if \( \pi_G (1 - \pi_G) < \pi_B (1 - \pi_B) \), it follows that the derivative w.r.t. \( q \), which is negative in both techniques, is numerically smaller in technique \( G \) than in technique \( B \). At the point where the two techniques are equally good for the borrower, the group liability will make \( G \) more advantageous than \( B \). As a consequence, the overall rationing \( e_{t-1} \) cannot increase and will typically increase.

Summing up, we have the following result.

**Theorem 1.** Suppose that \( \pi_G (1 - \pi_G) < \pi_B (1 - \pi_B) \) for all types \( i \). Then introduction of a small fraction \( q \) of liability for the other borrowers in each group will not decrease, and in some cases increase, the amount of investment \( e_{t-1}' \) carried out at time \( t - 1 \) by generation \( t \) and thereby increase overall welfare.
6. References


