Cheap Talk With Global Unknown Misalignment

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Abstract

The present study analyzes a cheap talk game engaged by two players, where one is endowed with some information and transmits it to the other. Communication about such information state is performed with costless messages and no effort in retrieving the information. Both players have a personal type of preferences over the application of the information state into an action. However, each one knows only her own type, while the others is unknown. Therefore, the receiver and the sender of the information state do not know the real preferences difference. Both players utility depend on the action taken at the end of the game by the receiver, where such action is based on the information transmitted. In the model, it is included a communication stage before cheap talk is performed, where the players signal each other their individual types, with which the game proceeds in the style of Crawford and Sobel (1982) model. The results of the proposed game are based in the ex ante expected utility of the players (before the sender is endowed with the information state knowledge), and they show that whenever the players' distribution of preferences satisfy certain conditions, informational communication is attainable even when the real misalignment is large enough to prevent it. In other words, informational communication is always feasible, if there exists the opportunity to engage it.

1 Introduction

The present study considers the interaction of two players. One of them wants to perform an action, which is based on an unknown information. For that reason, she asks for it to another player, who is naturally endowed with it. The way to transform the information into an action is commonly known, and depends on the information state and the personal preference of the player. Therefore, both players may present different preferred actions, which they would like to perform given an information value. However, only the uninformed player can take the ultimate action in the game. But this final realization depends on the information transmitted, with which the informed part would attempt to induce the receiver to perform her preferred action.

These kinds of models are called cheap talk games, because the messages sent are costless during the communication process. I focus on the situations where the players are misaligned, in the sense that they have different preferences about how to transform the information into an action. Moreover, I assume that the players know that they are misaligned, but they do not know how much, since they only know their own type of preferences. This fact is the main characteristic of the model I develop in this study.

Both players have the same function that generates an action, I assume that the relationship of the arguments of such function is additive. The only arguments on it are the information state and the specific preference or personal type. Using the ex ante (before the exposure of the information state to the sender) expected utilities of each player, I will prove that the game proposed can be structured as a static game with imperfect information. The solution is characterized for each player strategies, about how they signal their types. Therefore, it is a condition of the game, that after the players signal each other simultaneously their types, the continuation game via cheap talk is constructed using them exclusively. Meaning that the receiver will only listen messages about the information state, which she expects to hear.

The results show the existence of a pooling equilibrium, which means that despite their true types, there is a dominant strategy for signaling types. This unique strategy increases their ex ante expected utility functions. Also, there exists a hybrid equilibrium possibility for certain cases, when it is expected that the counterpart will play her dominant strategy with null probability. These results are constructed based on the assumption, that the sender possible types are higher than the receivers'. This assures the misalignment to be always positive, which means that the sender will have a higher preferred action than the receiver's. The equilibrium is constructed for the cases where among their type distributions, there is the possibility to incur in at least one informational communication[1]. I restrict the attention to distributions with two possible types per player.

This pooling equilibrium shows the willingness of the players to lie about their types, this in order to produce a more informational communication. Ex ante, it is always preferred by both players to have the most aligned communication, which may not be feasible using their true types. Whether the type signaled suits better the players utility (given a specific value of the information state) is to be studied in an ex post analysis, which it is not conducted here.

As remarked by Sobel (2010), while examining ex post situations, there are three associated problems with selecting the solution in cheap talk games. Multiple off the equilibrium path solutions, multiple meaning of messages and multiple equilibria associations between state information and actions taken by the receiver. Considering only the ex ante analysis allows to circumvent these problems, or at least to deferred them to another type of analysis, where is to be included refinements to select the solution among multiple equilibrium. The objective here, however, is to study the behavior of the players, when they signal simultaneously their types to each other. If both players commit to believe their types and they signal their types truthfully, the communication game converges exactly to the original Crawford and Sobel (1982) model. However, if the types are signaled strategically, it is shown, that there would be always informational communication, because the players would act as if they were as much aligned as they can, but the communication quality may decrease for a given realization of the information state.

The analysis is organized as follows, Section 2 exposes considerations regarding the cheap talk model and the multi stage kind of games, in Section 3 I perform a literature review of cheap talk games and studies that refer to the case where the receiver is unbiased and ignorant about the sender bias and the information state, while the sender signals the information realization

[1] Informational communication is referred to the situation encountered when the equilibrium strategies allow higher utility than in the case where no communication about the information state is performed.

but there is not bias disclosure, in Section 4 the game is formally presented, in Section 5 I characterize the theoretical approach of the equilibrium of the model proposed, Section 6 is devoted to illustrate the findings with an example and Section 7 concludes.

2 Considerations

The solution to the game known as cheap talk is credited to Crawford and Sobel (1982) and to the earlier versions of Green and Stokey (2007). Nevertheless, the focus of this study is based on the former's, where an information transmission model with two players is developed, one of them is privately informed about the information state of the world, while the other is endowed with the power of performing an action. Such action is performed given the knowledge of the information transmitted, passed directly by the informed part via costless messages. From here, the informed player is called "the sender" and her counterpart "the receiver".

The sender knows that the message she is going to transmit will affect her own utility via the action the receiver might take. The conflict in this game lays on the fact, that the sender and the receiver want different uses of the information as action. Typically, as it will be shown in the literature review, it is assumed that the receiver is unbiased, and the sender presents a positive bias towards any kind of action for a given information state. This fact implies that the latter prefers higher action realizations for a given information state. Given that both players know the magnitude of the bias, the receiver cannot expect to receive the exact information he is looking for, since she knows the sender want to persuade her to perform a different action.

Having the basic model established, it is worth to notice, that this setup involves no ex ante commitment from the receiver, in order to chose an action from a specific sub set of actions. Moreover, there are no monetary transfers for consulting the sender, and communication is costless. Nevertheless, there exists an endogenous cost for the sender to exaggerate, especially while signaling high messages, this is because the bias is known, and the receiver will tend to alleviate its effect after hearing the inflated signal.

From here, it is clear that there exists a misreporting incentive on the side of the sender, which conducts the proper transmission of the actual state of information to become noisy. This distortion would increase whenever the sender is highly biased or less aligned from the receiver. Moreover, given the solution Crawford and Sobel (1982) proposed, there exists an specific minimum level of misalignment in each game, where informational communication via cheap talk is attainable. However, for larger biases exists only babbling equilibrium[2]. Moreover, babbling equilibrium is always a solution possibility, even when informational communication is attainable.

The solution proposed is based in the Bayesian Nash equilibrium of Harsanyi (1968) given the incomplete information nature of the game. Since there is no verifiability of the messages, but the bias is known, the solution can be characterized as a partition equilibrium, where such

[2]When referring to babbling equilibrium I mean the equilibrium that provides the same utility as if communication were not even engage.

specific result depends on the magnitude of the misalignment[3]. The message space is partitioned N times depending the bias size. While the misalignment tends to zero, N tends to infinity, then, the information communication via cheap talk is more accurate. If $N \to 1$ caused by a large bias that exceeds the value of a threshold, the communication tends to a babbling equilibrium, where the sender is signaling the whole messages set, which is already known by the receiver, and then, communication is practically useless[4].

When common language among the players can be set, and the message space is partitioned in N parts, the model assures that there exists a best strategy for the sender after knowing the true information state. This is characterized by signaling one of those partitions. In that case, she could signal "the information state is in $[x,y]$", where x,y are in the information space and in the message space, and correspond to the boundaries of a partition. This signal increases the sender utility more than reporting other partition. Given the common knowledge of the bias, the receiver can expect to obtain a message like that and to believe it, because she knows that it includes the true state of information and the sender preferred action. When a common language is not set, the message is abstract and is open to multiple interpretations.

Because the game timing requires the emission of a message, followed by the receiver taking an action, the model is a kind of dynamic game. By definition, there should always be off the equilibrium path results. Moreover, it can be seen that whenever a bias magnitude allows certain number of partitions $N \geq 2$, there are multiple equilibrium. This is explained given the existence of babbling equilibrium for any amount of misalignment. Hence, this implies that there is more than one partition that she could choose from, in order to signal the information state, to induce the same receiver action.

In order to understand the behavior of strategic players, I allow them to signal each other their misalignments, with which the total bias can be computed. This approach implies to incorporate to the analysis another signaling period before the communication about the information is performed. During this initial stage, both players are able to communicate costless and simultaneously. To perform such analysis, it is a requirement to refer the solution to the class of strategic and dynamic games with imperfect information.

The strategic dynamic games with imperfect information are a compilation of consecutive continuation games, one consequence of the previous, each of them can start with a singleton or not. In a dynamic game, a history is the compilation of actions and strategies that lead to certain point in the game, where the player, whose turn to play is, performs her move based on the beliefs she has about the future moves of the other player, in other words, she solves the continuation game to decide which action to take next, and updates her beliefs about the imperfect information she has given the actions the other player has taken until then. In order to name a continuation game a proper sub game, the player has to know the history (past moves) where she is in the game. If the point she stands at is a singleton, the continuation game is a

[3] I treat the misalignment as the difference of the players preferred actions, which difference is the bias as treated in most of the literature.

[4] It is useless, because babbling equilibrium does not provide further hints, about where the true information state is, and equals the situation before engaging communication, where no information is transmitted.

proper sub game already, but whenever it is not, she should be able to assign a probability to each singleton that composes the history[5].

The solution to this type of games is given the Perfect Bayesian Equilibrium criteria. Seen from another perspective, it can be said that at each player history, the continuation game is solved with the Bayesian Equilibrium criteria developed in Harsanyi (1968). The game that I propose satisfies the conditions to be solvable with the Perfect Bayesian criteria, and it nests the original cheap talk game as a continuation game after the type signaling stage.

The fact that the first period of signaling involves a simultaneous move of the players, implies that the ex ante expected utilities of the original cheap talk can be used as payoffs, whenever it is possible to assign them a proper belief of happening. Therefore, it is straightforward to construct a game that is of static imperfect information to solve the complete game, if it is not to be analyzed the case where the sender is not yet endowed with the realization of the information state.

As a condition to find the strategies in equilibrium, it is necessary to assume that after signaling their types, the cheap talk game that continues will be performed with the misalignment signaled. Another important remark is that the game is solved only for ex ante situations. Therefore, the model does not select the partition to be played in equilibrium given the realization of a information state, which is required in ex post situations. Moreover, I assume a naive behavior of the players, exclusively while they form their strategies carried during the information transmission stage (the last stages). That means that they take the signals exchanged in the first period as their true misalignment.

3 Literature

3.1 About General Cheap Talk Games

As mentioned before, cheap talk games have been a prolific theme in information communication and signaling research. The simplicity of Crawford and Sobel (1982) approach is notorious, and together with the leading quadratic loss function example, this model has established itself as a workhorse in this research area. Moreover, the equilibrium is worked through intuitively, making its approach fairly easy to employ with differential equations. Consequently, numerous researchers have contributed to the literature with various considerations, which enriched the model in different situations.

Given the basic specification of the model, solutions that improve the common welfare were proposed, which add several players, either senders (Gilligan and Krehbiel, 1987; Krishna and Morgan, 2001; Battaglini, 2002; Ambrus and Takahashi, 2008) or receivers (Farrell and Gibbons, 1989), in order to compare information, and consequently reduce the misreport degree. These approaches have proved to be useful, and enlarge the possibility of revelation. These results hold even when the biases are larger than in the basic model.

[5] A proper definition of a Perfect Bayesian Equilibrium can be found in Fudenberg and Tirole (1991) for example.

Adding another consecutive stage, where cheap talk is played again by the same players was considered by Krishna and Morgan (2004) as another approximation. The repetition of the game allows players to update their set of strategies efficiently in the following stage. This efficiency occurs because they are able to update their believes in the next stage. All of these extensions mentioned before improve the Pareto allocation relative to the Crawford and Sobel (1982) original findings.

Approaches over the enlargement of the level of dimensionality of the game have been proposed in various perspectives. For example, Chakraborty and Harbaugh (2010) studies multidimensional cases, where dimensions are referred to objects (situations or subjects), which the information about them is comparable. The sender, in this type of game, proposes suggestions regarding the information state of the subjects comparing them. The receiver after hearing the suggestion performs an action involving the subjects. This study employs an unconventional mathematical method to find the equilibrium. The bias is characterized as the form of the sender's utility function, where the subjects are a linear or non linear combination in the utility function. The shapes of the indifference curves (specially the senders ones) are important, which evokes a limitation bounding the functionality of this approach. This model only works with quasi convex utility functions, which means that games involving senders, who might perceive a risk premium, have a feasible solution. Such equilibrium happens even when the sender is highly biased towards a specific action about the subjects. However the limitations, it exploits a not so popular fact, which is the characterization of the bias as the utility function shape, and not as a value as in the basic model. The sender, as a risk lover, fully reveals her preferences, because of the common knowledge of her utility function. Another important remark is about the sender state independence, which means that although she knows the true state of the information, her utility only depends on the realizations performed by the receiver. These facts together with the multidimensionality are the key components of the game. Contrasting to Crawford and Sobel (1982)'s model where the players utility functions depend on the action performed by the receiver and on the specific value of the information state.

On the other hand, Levy and Razin (2007) demonstrate that even in multidimensional models, informative equilibrium typically disappears when there is sufficiently large conflict of interest. In this model are two messages regarding two different subjects, and two respective actions decided. However, informational communication is attainable for one of the dimensions where there is no conflict. These findings contrast Chakraborty and Harbaugh (2010) that ensure that communication is attainable and preferable by the two players even with large disagreement. Levy and Razin (2007) focus their study in spillovers of information on one dimension over another.

Delegation is a refinement of the game, which has become one of the main areas of research. Occurs when the receiver gives to the sender the power of deciding what action to take. Nevertheless, the receiver restricts the set of possible actions, from which the sender chooses her preferred one. This literature evolves strongly after Holmström (1979, 1984)[6]. This kind

[6]Holmström (1979) is a paper based on the forth chapter of the author's unpublished Ph.D. dissertation in Stanford University.

of setup was analyzed with existence and non existence of monetary transfers. Melumad and Shibano (1991) explore the differences in cases of delegation and no commitment in a setup of no transfers. Alonso and Matouschek (2008) generalize the solution of delegation without transfers, allowing general distributions and broader families of utility functions. As Dessein (2002), they conclude that delegation is optimal, and better whenever the players are more aligned. However, a solution still might exist for higher biases in the form of a non continuous set as studied by Szalay (2005) and Alonso and Matouschek (2008) in different model settings. Going even further to restrict the set of messages and increasing the gap in the middle, one could delegate the suggestion only for the extreme points of the set as proposed by Kawamura (2011).

Another problems arise in cheap talk, since communication can be performed through various means and in different setups. For example, there is the case of cheap talk in organizations, where hierarchies communicate vertically. In the simplest case, Liang (2012), in the spirit of Dessein (2002), has employed delegation in a setup of three levels of vertical communication, where delegation is a possibility only from one level down to the immediate lower. The findings suggest the reason why top levels should preferably position bottom level workers that are strictly more aligned to them. A more complex game focus on studying the communication within groups and between them. Alonso et al. (2008) examine the case where senders in each group observe different pieces of information, which makes difficult to achieve coordination between groups. As a result, decentralization of the groups outperforms centralization when the group receivers (top hierarchy within the group) incentives are sufficiently aligned. However, it is also shown that vertical communication with a higher hierarchy (each group leader with a headquarters leader) is more efficient than horizontal communication among group receivers. This because, group leaders share more information with an unbiased headquarter leader than with each other.

Focussing in Dessein (2002) results, it is important to remark that, given the usual workhorse model of Crawford and Sobel (1982), delegation is preferable to communication. This happens whenever the set of messages has more than one partition in cheap talk terms. In contrast, letting all values constant, it is shown that communication can be preferable to delegation when the information state set can be reduced. This reduction allows finer partitions than in the original set, and therefore the receiver can perform more accurate decisions. However, in this last environment, there exists still noise in the sender messages.

The research in this area has grown because of the improvement delegation means to the information communication. Specially for the receiver[7], who is the one that restricts the message set. Delegation works as a convenient tool for avoiding conflicts in higher information states.

In various models, it is assumed that the sender do not incurs in any cost for retrieving the information state. However, for example Che and Kartik (2009) study the endogenous information retrieval situation, where it is claimed that whenever there is a high bias, less informational communication exist. Nevertheless, this situation creates sender incentives for information ac-

[7]Delegation lowers benefits for senders when they are more biased.

quisition. As a side note, it can be seen, that these results contrast some of the ones found in behavioral economics literature. Bénabou and Tirole (2003) propose theoretically, that there exist incentives that can harm the performance of a task. It is an interesting avenue to analyze, whether delegation is a sufficient motivator for the sender to obtain the information required. For Che and Kartik (2009), the incentives for retrieving the information are self enforced, because exerting more effort allows the sender to perform persuasion and avoid prejudice. Moreover, the study shows that delegation can be demotivating because it eliminates the sender's need to persuade and avoid prejudice.

This delegation models (viewed as well as mediation problems) are shown as optimal deterministic mechanisms to confront the problem given information asymmetries and misalignment of preferences. Randomizing mechanism is also possible. Kovác and Mylavanov (2009) study this fact concluding that deterministic delegation dominates the stochastic approach.

The other avenue of research on cheap talk games has been performed towards developing refinements respect the equilibrium criteria, in order to potentially reduce the set of possible optimal strategies. In studies of Farrell (1986, 1988, 1993); Matthews et al. (1991), it is intended to look for an interpretation of messages where there are many induced equilibrium because of the messages meaning and intended meaning problem. Farrell (1993) proposes a refinement called the neologism proofness. However, other perspective about this topic could be seen as well as a problem of message quality, where the suggestion sent is perturbed stochastically, as shown by Blume et al. (2007). In other studies, the problem has been analyzed with a certain degree of agnosticism about the reason why multiple equilibriums exist. Chen et al. (2008) propose a selection of equilibrium criteria, NITS, that is supported in the Crawford and Sobel (1982) monotonicity condition, where such induced result will not be neologism proof in the Farrell (1993) sense. This approach helps to reduce the set of multiple equilibriums, and it is based on the idea that when the sender observes low realizations of the information state, and has a positive bias, she has no incentive to signal a message that is below the state observed.

Ultimately, the solution criteria and equilibrium selection are based mainly on the model specifications proposed in Crawford and Sobel (1982). Most of the literature is based in the positive biased sender. However, Gordon (2010) studies the cheap talk game through another avenue, and grants solutions for all the possible directions the bias can take. Moreover, Gordon (2011) provides an equilibrium selection that satisfies an iterative procedure, and does not depends exclusively in the monotonicity condition of Crawford and Sobel (1982). This approach does not need to assume any specific form of the utility function of the players. These analyses are performed with a supermodularity approach that is based in the ordering of strategies and information combinations in lattices.

All models cited until here are built on the assumption of common knowledge of the biases. Another direction of research is less prolific, and investigates cheap talk models where an unbiased receiver presents uncertainty about the sender bias.

3.2 About Cheap Talk Games With Bias Uncertainty

One of the first approaches, and the most cited study when examining the uncertainty of the sender's bias is Dimitrakas and Sarafidis (2005). This analysis shows the strategical behavior of a sender who signals the information state, such message is based in the posterior belief of the receiver about the sender's bias and the information state. They assume that both uncertain variables are continuously distributed. They conclude that this setup allows informational communication even when the sender is more biased than in Crawford and Sobel (1982) model.

This last study helps to embody Li and Madarász (2008) work, where they compare situations where the sender performs or not the disclosure of her bias. This model allows for the sender to be biased positively or negatively. Their results point that non disclosure is preferable, however, compulsory disclosure provides meaningful information transmission.

There exist approximations, such as Morgan and Stocken (2003), in which they analyze a situation where the sender with unknown bias performs stock recommendation to an uninformed receiver. They conclude that aligned senders can send credibly unfavorable information but cannot signal credibly favorable information for the receiver. This result is based in the construction of this specific stock recommendation model. However, the intuition is clear in terms of the basic cheap talk model, where lower states are sent more credibly than higher states specially when the sender bias is unknown.

Although these approximations show the possibility of model a game where the sender bias is unknown, it has not been proposed a model where even the sender has uncertainty about the total difference of preferences in the game. In the present exposition, I perform the analysis for the case where both players know their preferred information application, but they are ignorant regarding the complete misalignment that exist in the game. In this case, I perform an extension of the Crawford and Sobel (1982) model when this situation is met. As it has been remarked, the knowledge of the bias is important for the game to have a solution, since cheap talk information is unverifiable. Therefore, I allow in my model the players to signal each other preferences in a simultaneous strategic communication pre engaging cheap talk with the misalignment that resulted signaled in the first period.

4 Model

The model I propose is similar to Crawford and Sobel (1982) model. Both models are based in the information transmission problem, in which the players action preferences are different, and they present the same assumptions regarding the players utility functions. However, I make explicit the difference of the players preferred actions, allowing the receiver to have a personal bias as the sender does. Typically, my model converges to Crawford and Sobel (1982) model for additive action functions. This case is studied here extensively in order to compare them.

Assume that both players are privately endowed with a type. For the players in the game, there exists an action function $a(m,t)$, which is a transformation of the information state m and the player's type t into an action. Without loss of generality, the type as defined here is the bias that each player has, and it determines the way the information is transformed into an action, which is characterized as the function a. I assume that this action function is an additive transformation, meaning that $a(m,t) = m+t$.

Throughout the model, it is assumed that the personal types of the players are private information. Since the knowledge of them is vital for the game to be solvable, the players exchange simultaneously signals about them. Therefore, in the first period, the players signal each other their types, in the second, the sender signals the information state with the message space partitioned given the types signaled, and in the last period the receiver performs a final action. In these last periods the players assume that the types signaled are the true types. In the period zero, the nature decides a value of information, a type for the sender and a type for the receiver. The solution of the game is based in a Perfect Bayesian Equilibrium.

The conflict in this game is produced by the difference of types the players have. As mentioned before, this is unknown for both players who are only certain about their own type. Each player type is Bernoulli distributed and I assume that the types in the sender distribution are strictly larger than all possible receiver's types. This allows positive misalignment in all situations, and it is the only certainty the players have about their action preferences differences.

Throughout the game, the utility functions are common knowledge for both players. The idea how they are constructed is based on what the players are interested on. They both want an action to be performed by the receiver at the end of the game, after hearing the sender's signal. Nevertheless, they both want specifically their own preferred action to be played at the end of the game. Then, their utility functions are composed by the difference of the receiver's action taken at the end of the game and the player's preferred one. Typically, more utility would be attained whenever this difference is minimized.

Formally, the utility function for player i measures the distance between the action that is going to be performed by the receiver ($a(y,t^R)$) after hearing the senders message, and the personal preferred action of player j ($a(m,t^j)$) given the information m. Then, the utility is defined as $U^j(a(y,t^R) - a(m,t^j))$, where $j = R,S$ are the receiver and the sender respectively. The type of each player is $t^i \in T$ where $T \in \{\underline{t}, \overline{t}\}$, the information state is $m \in M$ where $M \in [0,1]$ is a compact set uniformly distributed, the action function is defined as $a : M \times T \to A$ where $A \in [\underline{a}, \overline{a}]$ is a compact set. The only argument that is not yet defined is y and requires special attention to be given in the next paragraph. I assume that the utility function has a bliss point in zero, the reason why this is the case is because the player i reaches her highest utility when the distance between $a(y,t^R)$ and $a(m,t^j)$ is null, whenever this distance increases, the utility decreases. The assumptions regarding the derivatives of the utility function and the action function are presented in the formal definition.

Definition 1 (Action function)**.** *The action function is defined as $a(m,t)$, for an $m \in M$ and $t \in T$, where $a : M \times T \to A$ produces a unique a and presents $\partial a/\partial m > 0$ and $\partial a/\partial t > 0$. Moreover, the action function is characterized to be additive $a(m,t) = m+t$*

Definition 2 (Preferred action)**.** *The preferred action for player j, where $j = R,S$, is defined as $a(m,t^j)$ for a given $m \in M$ and $t^j \in T$. Then $a(m,t^j) = m+t^j$.*

To allow a better understanding of the variable y, consider the following cases. Whenever $t^R = t^S = t$, for a given m both players have the same preferred actions $a(m,t^R) = a(m,t) = a(m,t^S)$, which is a result of Lemma 1 defined in the next section. If both players are aware about this full alignment, then the sender would like to maximize her utility function by sending a message $n = m$. Therefore, the receiver would understand $y = n$, and she will execute the action that is preferred for both players $a(y,t)$, which is equal to both players preferred actions, thus the utility functions would reach the bliss point, and $U^i(a(y,t) - a(m,t)) = 0$ where $y = n = m$.

Whenever the types are different $t^R \neq t^S$, the sender cannot send a message n that equals the true state m, because a naive receiver would understands $y = n = m$, and therefore will take action $m + t^R$, which is different to $m + t^S$. When both players known that $t^R \neq t^S$, a non strategical sender would send a message $n = m + t^S$, which is her preferred action. A strategical receiver would know that the real information state is inflated additively by t^S, and she will understand $y = m = n - t^S$ correcting for the sender's bias. Hence, she will perform the action $m + t^R$ which is different to $m + t^S$.

However, none of these cases would happen in a strategical game where both players are rational. Moreover, how the game would finish depends on the strategical message sent and how the receiver will rationally understand it. Therefore, the variable y depends on n. This is the point where communication problems arise given the lack of a common language among players, because n is not what affects the utilities. What really matters is what it means to the receiver, and what she can understand about it. In other words, it only has influence changing the beliefs of the receiver about m through y.

With all the arguments exposed, I formally postulate the utility function and the assumptions behind it.

Definition 3. *The utility function for player j is $U^j(a(y,t^R) - a(m,t^j))$ measures the distance between the actions. It has a unique bliss point in zero, and it is assumed that:* $\partial U^j / \partial y < 0$, $\partial (U^j)^2 / \partial^2 y < 0$, $\partial (U^j)^2 / \partial y \partial m > 0$, $\partial U^S / \partial y \partial t^S > 0$, *where the double derivatives are calculated with the chain rule. With an additive action function the focus is on utilities of the form $U^R(y,m)$ and $U^S(y,m,t^S - t^R)$.*

In this framework with individual private information, the assumption about how expectations are formed is necessary. There are three variables that are not common knowledge, each players type and the information state. These variables are generated by their own probability distribution independently from each other. All the players know the distributions and the bounds the variables are enclosed in, and they are aware about the fact that they are all independently distributed. I define $f(m)$ as the uniform distribution of the information state $m \in M$. $\gamma(t^S)$ is the probability mass distribution for the sender and $\delta(t^R)$ is the receiver's, these last two are Bernoulli distributed.

The game is constituted by signals and responses. The signals exchanged in the first period $\tilde{t}^R$ and $\tilde{t}^S$ follow this rule:

1. For the sender case, each $t^S \in \{\underline{t}^S, \overline{t}^S\}$, $\int_{\tilde{T}^S} q(\tilde{t}^S|t^S)d\tilde{t}^S = 1$, where the set $\tilde{T}^S = \{\underline{\tilde{t}}^S, \overline{\tilde{t}}^S\}$ is the set of feasible signals, and if $\tilde{t}^{*S}$ is in the support of $q(\cdot|t^S)$, then $\tilde{t}^{*S}$ solves

$$\max_{\tilde{t}^S \in \tilde{T}^S} \sum_{t^R \in T^R} \sum_{\tilde{t}^R \in \tilde{T}^R} U^S(a(y(n^*), \tilde{t}^R) - a(m, t^S))q(\tilde{t}^R|t^R)\delta(t^R)$$

$$\max_{\tilde{t}^S \in \tilde{T}^S} \sum_{t^R \in T^R} \sum_{\tilde{t}^R \in \tilde{T}^R} U^S(y(n^*), m, t^S - \tilde{t}^R)q(\tilde{t}^R|t^R)\delta(t^R). \tag{1}$$

2. For the receiver case, each $t^R \in \{\underline{t}^R, \overline{t}^R\}$, $\int_{\tilde{T}^R} q(\tilde{t}^R|t^R)d\tilde{t}^R = 1$, where the set $\tilde{T}^R = \{\underline{\tilde{t}}^R, \overline{\tilde{t}}^R\}$ is the set of feasible signals, and if $\tilde{t}^{*R}$ is in the support of $q(\cdot|t^R)$, then $\tilde{t}^{*R}$ solves

$$\max_{\tilde{t}^R \in \tilde{T}^R} \sum_{t^S \in T^S} \sum_{\tilde{t}^S \in \tilde{T}^S} U^R(a(y(n^*), t^R) - a(m, t^R))q(\tilde{t}^S|t^S)\gamma(t^S)$$

$$\max_{\tilde{t}^R \in \tilde{T}^R} \sum_{t^S \in T^S} \sum_{\tilde{t}^S \in \tilde{T}^S} U^R(y(n^*), m)q(\tilde{t}^S|t^S)\gamma(t^S). \tag{2}$$

Where the associated values of y and n^* are calculated for every possible misalignment signaled with the following rules:

- For each $m \in [0.1]$, $\int_N q(n|m)dn = 1$, where the set N is the set of feasible signals and if n^* is in the support of $q(\cdot|m)$, then n^* solves

$$\max_{n \in N} U^S(a(y(n), \tilde{t}^R) - a(m, \tilde{t}^S))$$

$$\max_{n \in N} U^S(y(n), m, \tilde{t}^S - \tilde{t}^R). \tag{3}$$

- For each $n(\tilde{t}^S, \tilde{t}^R)$, $y(n)$ solves

$$\max_y \int_0^1 U^R(a(y, t^R) - a(m, t^R))p(m|n)dm$$

$$\max_y \int_0^1 U^R(y, m)p(m|n)dm. \tag{4}$$

where $p(m|n) = \frac{q(n|m)f(m)}{\int_0^1 q(n|m')f(m')}$.

At this point is necessary to highlight some facts about the signal rules of the game. The receiver presents in her maximization problem (2) no dependency on her own type. However, it does depend on her type signal indirectly through the message sent by the sender. In contrast, the sender's routine (1) depends on her actual type, meaning that how much is she going to be able to go beyond her true type depends on the expected misalignment respect her true type. The receiver's strategy only depends in maximize her utility given the signal the sender would emit, without any regards over her own type.

In order to summarize, I assume that player i utility depends on the difference of ultimate action taken by the receiver and the player's i preferred action. I allow both players to be biased (to have a type) towards a preferred action. Both players types are only private information, and signaled to each other strategically before communicating the signal about the information, with which the ultimate action is taken. Both players commit to believe the signaled types as their true types with which cheap talk is conducted. The receiver in my model understands the signal about the information state with which an action is to be taken.

5 Equilibrium

In this section, I characterize the cheap talk equilibrium for the model proposed in the previous section. The main focus is to find the preferred strategy of the sender in the first period of communication, where the types signals are exchange simultaneously. The condition to find the strategies in equilibrium is to focus on the ex ante[8] utility functions, in order to obtain the attainable pay offs for the first period. The ex post analysis is not performed; nevertheless, certain considerations are made while conduction the example in the next section.

In the first part of this section the expectation and the probability characterization will not be taken in consideration. Whenever it will be necessary, it will be added explicitly. The analysis begins studying the players behavior while communicating the information state, which equilibrium is characterized by Crawford and Sobel (1982). Next, the exposition is focused on the strategies while signaling the types. The reason for proceeding this way is because how the players signal their types is a consequence of the cheap talk equilibrium. Since this model can be seen as an extension of Crawford and Sobel (1982), I use their theoretical approach and findings to give shape to my model. Therefore, I collect their most important findings for which the proofs are already exposed in Crawford and Sobel (1982). However, the intuition about such the theoretical approach will be provided.

Equally as Crawford and Sobel (1982), it is allowed all signals n to be compiled in the group $\bar{N}$ if induce $\bar{y}$ whenever $\int_{\bar{N}} q(n|\bar{m}) > 0$, then $\bar{m}$ induces $\bar{y}$, such selection of $n \in \bar{N}$ is signaled if $U^S(\bar{y}, \bar{m}, t^S - t^R) = \max_{y \in Y} U^S(y, \bar{m}, t^S - t^R)$. Because of the assumption $\partial(U^S)^2/\partial^2 y < 0$ the utility function can take at most two values for a given y, and $\bar{m}$ induce at most two y in equilibrium.

As there are preferred actions for the players, there are also preferred information understandings y for each one.

$$y^S(m, t^S - t^R) = \arg\max_{y \in Y} U^S(y, m, t^S - t^R) \tag{5}$$

$$y^R(m) = \arg\max_{y \in Y} U^R(y, m) \tag{6}$$

In equation (6), $y^R(m)$ only depends on m, because as defined above, the utility function measures the distance between two actions. Whenever the argument t^R is fixed, the difference is just due to m, which is a direct conclusion from Lemma 1. Therefore, the understanding

[8]As remarked before ex ante and ex post are referred to the receiver endowment of the information state.

of the information just depends on the value of the information. For the sender, (5) depends on both players misalignment. As noted by Crawford and Sobel (1982) because of the utility function assumptions, $y^S(m,t^S-t^R)$ and $y^R(m)$ are well defined and continuous in m, since this is continuous and bounded.

Lemma 1. *If the preferred actions of the players are different, $a(m,t^R) \neq a(m,t^S)$, it is because the misalignment $t^R \neq t^S$.*

Proof. By contradiction, assume the action function produces $a(m,t) = v$ for a given m and t, if there exists an $a(m,t') = v$, it must the case that $a(m,t) = a(m,t')$, thus $t = t'$, which is a contradiction. Therefore, for a given m and $a(\cdot,\cdot)$, if $a(m,t) \neq a(m,t')$, it must be the case that $t \neq t'$ □

Lemma 1 formally shows the claim that the preferred actions are equal whenever the types are the same. This conclusion is trivial when the action function is additive.

Lemma 2. *If the players' types are misaligned, $t^R \neq t^S$, then the preferred understandings of the information of both players differ, $y^R \neq y^S$.*

Proof. By contradiction, assume $t^R = t^S = t$ then because of Lemma 1 $a(m,t^R) = a(m,t^S) = a(m,t)$, therefore according to (5) and (6)

$$y^S(m) = \arg\max U^S(y,m,t-t) \tag{7}$$

$$y^R(m) = \arg\max U^R(y,m) \tag{8}$$

then for a given m, $y^S(m) = y^R(m)$, which is a contradiction. In this situation, the utility functions reach the unique bliss point with respectively $y^S(m)$ and $y^R(m)$. This case happens uniquely when $t^R = t^S$, because it is the situation where the two players reach the unique bliss point.

Therefore, for the other cases where $t^R \neq t^S$, there exist $y^S(m,t^S-t^R)$ and $y^R(m)$, and it must be the case that even when one of the types is zero, the other has to be different to zero ($t^R \neq t^S$), then, $y^R \neq y^S$ holds, and holds whenever none of the types are zero and satisfies $t^R \neq t^S$. □

The intuition behind Lemma 2 establishes that; the only situation when the receiver is going to understand a signal about m exactly as the signal suggested is when the types of both players are equal and common knowledge. The next lemma was postulated by Crawford and Sobel (1982) and its proof is based on theirs.

Lemma 3 (Lemma 1 and proof in Crawford and Sobel (1982))**.** *If $y^S(m,t^S-t^R) \neq y^R(m)$ for all m, then, there exists an $\varepsilon > 0$ such that if u and v are understandings induced in equilibrium, $|u-v| \geq \varepsilon$. Further, the set of understandings induced in equilibrium is finite.*

Proof. Assume the understandings of the receiver under a signal sent in equilibrium are $u < v$, from which the sender reveals a preference for one of those (the preference for one must be true since the $u \neq v$). Because of continuity, there must be a $\bar{m} \in [0,1]$ for which the utility of the sender is the same for either u or v $U^S(a(u,t^R),a(\bar{m},t^S)) = U^S(a(v,t^R),a(\bar{m},t^S))$.

$$u < y^S(\bar{m},t^S - t^R) < v \tag{9}$$

Because $\partial U^j/\partial y < 0$ and $\partial (U^j)^2/\partial y \partial m > 0$ the preferred understanding of the sender $y^S(\bar{m},t^R,t^S)$ is between u and v, and because $\partial (U^j)^2/\partial y \partial m > 0$, the understanding u is not induced by any $m > \bar{m}$, and v is not induced by any $m < \bar{m}$. This produces:

$$u \leq y^R(\bar{m}) \leq v \tag{10}$$

The inequality holds strictly when Lemma 2 is satisfied. When $t^R \neq t^S$ then $y^R \neq y^S$ for any m, then there is $|y^R - y^S| \geq \varepsilon$ for all $m \in [0,1]$. From (9) and (10) it must follow that $v - u \geq \varepsilon$ where the set of understandings is bounded by $y^R(0)$ and $y^R(1)$. □

This Lemma refers to the fact that because Lemma 2, in equilibrium it is not possible for the sender to induce his preferred understanding when the types are different. The result is a noisy communication, where the produced understanding is not invertible, and the true information state is unattainable. The lemma shows as well that there is only a finite set of understandings induced in equilibrium, since there is only a reduced set of information states with which those are feasible. In this case the inclusion of the action function does not change the results of the Crawford and Sobel (1982), since the assumptions of the utility functions are the same. Moreover, the lemma suggests that any equilibrium is a partition equilibrium.

To illustrate this lemma, assume an utility quadratic loss utility function in equilibrium that satisfies the assumptions defined.

$$U^R = -(y-m)^2 = x, \tag{11}$$

The understandings u and v induced in equilibrium are:

$$y = m \pm \sqrt{x}. \tag{12}$$

From here, it can be seen already that whenever $x \neq 0$, then, the difference between u and v is some ε. Now it is necessary to find the value of m, for which the sender and receiver understandings are in equilibrium.

$$U^S(u,\bar{m},t^S - t^R) = U^S(v,\bar{m},t^S - t^R) \tag{13}$$

Which yields as result $\bar{m} = (v+u)/2 - (t^S - t^R)$. Getting the preferred sender understanding given $\bar{m}$, it is obtained $y^R = (u+v)/2$, and this satisfies equation (9). The preferred receiver understanding given $\bar{m}$ is $y^S = (u+v)/2 - (t^S - t^R)$ satisfies (10). In this last step, it is seen how the receiver corrects her understanding given the misalignment. Moreover, since an equilibrium has been computed here, it can be seen that whenever the types are different, any sender message can only produce two receiver understandings in equilibrium (12), also, the set of information state for which it has been generated is bounded. This result can also be founded in Li and Madarász (2008).

As in Crawford and Sobel (1982), I form the equilibrium as a partition equilibria. The essence of the equilibrium is based in the continuous partition of the message set defined in $[0,1]$ in N parts. The bins separating each partition are denoted in $a(N) = (a_0(N), a_1(N), \ldots, a_N(N))$, where $a_0(N) < a_1(N) < \cdots < a_N(N)$ with lower bound $a_0(N) = 0$ and upper bound $a_N(N) = 1$. Without loss of generality, for all $\underline{a}, \overline{a} \in [0,1]$, a pair of bins one besides the other satisfies $\underline{a} < \overline{a}$. A signal (or message) n regarding the information state is conformed by a pair of a, where one is next to the other. Basically, the receiver signals the neighborhood where the information state is.

Given a signal n, the receiver strategically performs her response, where she understands the signal given the probability of occurrence of the set of signals and the set of the information.

Definition 4 (Best response of the receiver)**.** *In expectation, the best response of sender's understanding given a sender's message $\underline{a}, \overline{a} \in [0,1]$ uniformly distributed is:*

$$\overline{y}(\underline{a}, \overline{a}) = \begin{cases} \arg\max \int_{\underline{a}}^{\overline{a}} U^R(a(y,t^R) - a(m,t^R)) f(m) dm & \text{if } \underline{a} < \overline{a} \\ y^R(\underline{a}) & \text{if } \underline{a} = \overline{a} \end{cases} \tag{14}$$

As a side note, it is important to note that the explicit receiver's best response just defined is based in the maximization of the conditional expectation of the utility. The nature of its derivation is based in the assumption that the information state is uniformly distributed. A deeper understanding of this result is given after the theorem that defines the existences of the equilibria.

Theorem 1 (Theorem 1 in Crawford and Sobel (1982))**.** *Suppose $t^R \neq t^S$ and Lemma 2 holds for every m. Then there exists a positive integer $N(t^R, t^S)$ such that for every N with $1 \leq N \leq N(t^R, t^S)$, there exists at least one equilibrium $(y(n), q(n|m))$, where $q(n|m)$ is uniform and supported on $[a_i, a_{i+1}]$ if $m \in (a_{i+1}, a_i)$.*

$$U^S(a(\overline{y}(a_i, a_{i+1}), t^R), a(m, t^S)) - U^S(a(\overline{y}(a_{i-1}, a_i), t^R), a(m, t^S)) = 0 \quad \text{for any } i = 1, \ldots, N-1 \tag{15}$$

$$y(n) = \overline{y}(a_i, a_{i+1}) \text{ for all } n \in (a_i, a_{i+1}) \tag{16}$$

$$a_0 = 0 \tag{17}$$

$$a_N = 1 \tag{18}$$

Further, any equilibrium is essentially of this class for some values of N with $1 \leq N \leq N(t^S, t^R)$

The proof is in Crawford and Sobel (1982), and holds in the present model since the assumptions regarding the signals and utility functions have the same assumptions and arguments as in the original work.

For clarity's sake, I present the intuition that leads to the equilibrium given the lemmas and theorem exposed. Both players form strategical response functions. Let the interval $[a_i, a_{i+1}]$ be defined as a step. Equation (15) is known as the sender's arbitrage condition, it shows the point where she is indifferent between two receiver understandings for different but consecutive steps. This last argument implies that there should be an a that is in both signals, typically the one that is between two signals dividing these steps, namely a_i for $[a_{i-1}, a_i]$ and $[a_i, a_{i+1}]$. Solving for this bin generates a well defined second order non linear difference equation, where (17) and (18) are the initial and terminal conditions. With this difference equation it can be found the steps defined by the bins $a(N) = (a_0(N), a_1(N), \ldots, a_N(N))$, which are in the set of $[0,1]$ given conditions (17) and (18). For a value of m, the sender can choose the step where the state lies in, which maximizes her utility given the receiver understanding (16) that would be induced with this message. The arbitrage condition assures, for values of $m \in (a_i, a_{i+1})$, this message is strictly preferred among the others, with the exception of values of m that are exactly equal to one of the bins, where, of course, the sender is indifferent between those two messages. This last fact is a consequence of the nature of the arbitrage condition.

The receiver on the other hand, maximizes her conditional expected utility given the signal received. This criteria is characterized in equation (4). The Perfect Bayesian Equilibrium states that at every continuation game, the strategies on the equilibrium path must be formed rationally, and they must be a Bayesian equilibrium by itself, where the beliefs are updated with Bayes' rule when possible. Therefore, without taking into account the first part of the game where the players signal their types, the receiver's best strategy is constituted as (4).

The receiver's beliefs updating, when receives a signal that is in the interval (a_i, a_{i+1}), are formed as follows

$$p(m|n) = \frac{q(n|m)f(m)}{\int_{a_i}^{a_{i+1}} q(n|m')f(m')dm'} = \frac{f(m)}{\int_{a_i}^{a_{i+1}} f(m')dm'}, \tag{19}$$

which replacing in the conditional expected utility yields

$$\int_{\underline{a}}^{\bar{a}} U^R(a(y,t^R) - a(m,t^R))p(m|n)dm = \frac{\int_{\underline{a}}^{\bar{a}} U^R(a(y,t^R) - a(m,t^R))f(m)dm}{\int_{a_i}^{a_{i+1}} f(m')dm'}, \tag{20}$$

which holds for uniform distribution of m, where $f(m) = 1$ when $m \in [0,1]$. The solution that maximizes this expectation is the understanding (16) as defined in Definition 4 for an $n \in (a_i, a_{i+1})$. Moreover, this result has to be a best response for all $n \in N_i$ where $N_i = n : y(n) = y_i$. This means that in the set N_i are all the signals that produce the understanding y_i when the sender sees an information state $m \in (a_i, a_{i+1})$. To satisfy condition (15), (17) and (18), the best response (16) to any signal $n \in N_i$ has to maximize the conditional expected utility:

$$\int_{\underline{a}}^{\bar{a}} \int_{N_i} U^R(a(y(n),t^R) - a(m,t^R))q(n|m)dndm = \int_{\underline{a}}^{\bar{a}} U^R(a(y(n),t^R) - a(m,t^R))f(m)dm \tag{21}$$

Where the conditional density integrates to the unity because its whole mass is uniformly distributed trough N_i. Moreover, since $N_i = n : y(n) = y_i$, $y(n)$ is constant over the integration range.

As seen until here, the model has a solution as partition equilibrium. Theorem 1 establishes the existence of a partition with $N(t^R, t^S)$ steps, which depends on the difference of the players' types. It is easy to see this fact from solving the difference equation formed with the arbitrage condition. As noticed before, such equation (15) depends on both players type difference given the additive nature of the action function. Moreover, it depends on the collection of bins a for which the difference equation is solved. This leaves the difference of types as the determinant for the number of partitions that occurs in equilibrium. Crawford and Sobel (1982) provides the formalization to this fact

Lemma 4 (Lemma 6 in Crawford and Sobel (1982))**.** *The maximum possible equilibrium partition, $N(t^S, t^R)$, is non increasing in $\sqrt{(t^S - t^R)^2}$, the distance between agents's preferences.*

The simplest partition equilibrium that could occur in the game occurs when the message set is divided one time. Theorem 1 proves it existence, along with all the other partition equilibrium that could occur depending on the misalignment. Moreover, Crawford and Sobel (1982) show that this equilibrium is non informative. It can be seen in equation (16), which is a result from Definition 4, that whenever the sender signals a step $[a_0, a_1]$, and those correspond to the message space bounds defined in (17) and (18), the receiver is in an equal situation as when she would be without communication. This fact is true, since by the model definition she knows already that the information state is in $[0, 1]$, she could compute the same result without communication.

The ex ante expected utilities are important for comparing different realization of the game. Whenever the information state is drawn and shown to the sender, the preferences over the equilibrium to be chosen depends on such realization. This occurs when more than one partitions are available. Therefore, a generalization for ex post (in this sense) case is a sensible matter and for the comparison's sake, the following analysis is performed over ex ante utility expectations.

Definition 5 (Expected Utility)**.** *Using similar calculations as in equation* (21) *the utility expectations are defined over the players types t^j and the signals about their types $\tilde{t}^j$. For a receiver type t^R who signals $\tilde{t}^R$ her expected utility ex ante is defined as:*

$$EU^R = \sum_{t^S \in T^S} \sum_{\tilde{t}^S \in T^S} \sum_{i=1}^{N} \left(\int_{a_{i-1}}^{a_i} U^R(a(y(a_{i-1}, a_i), t^R) - a(m, t^R)) f(m) dm \right) q(\tilde{t}^S | t^S) \gamma(t^S) \tag{22}$$

For the sender with type t^S and signaling $\tilde{t}^S_k$ her expected utility ex ante is defined as:

$$EU^S = \sum_{t^R \in T^R} \sum_{\tilde{t}^R \in T^R} \sum_{i=1}^{N} \left(\int_{a_{i-1}}^{a_i} U^S(a(y(a_{i-1}, a_i), \tilde{t}^R) - a(m, t^S)) f(m) dm \right) q(\tilde{t}^R | t^R) \delta(t^R) \tag{23}$$

Where $N = N(\tilde{t}^S, \tilde{t}^R)$.

When simultaneous type signaling is allowed before cheap talk, the game can be treated as a static imperfect game. The reason why this holds is based on the assumption about the players belief that the true misalignment is the one signaled. Therefore, the possible payoffs for each combination of type signals for each real players' types can form the game payoffs for every situation possible. This fact is true only if it is analyzed the ex ante case, since in the ex post situation, the multiple equilibrium existence does not allow an accurate analysis.

Theorem 2. *The multi stage signaling game with costless and unverifiable messages can be reduced to a static game with imperfect information where the payoffs are produced by the expected utilities ex ante the realization of the information state. In the first period, the players signal their types (preference about how to execute the information state) simultaneously. During the second period the sender delivers a message. During the final period the receiver performs an action. In the last two periods the game is conducted with the assumption that the players believe that the types signaled in the first communication period are the true ones.*

Proof. To prove the theorem it is needed to show that the game satisfies the Perfect Bayesian Equilibrium conditions. Then, the justification of why the game turns to a static game with imperfect information finishes the proof.

The PBE as postulated by Fudenberg and Tirole (1991) requires that in every player's point in the game that is not a singleton, the player is able to assign a probability to all the elements that are in such non singleton. This is guaranteed when the prior probability distributions of the types and information state are common knowledge for both of the players. Therefore, all the continuation games can be treated as sub games. Thus, the cheap talk model constitutes a proper sub game for every combination of types and type signals, and presents a unique solution for the ex ante expected utilities as shown in their definition for both players.

As seen in Lemma 4, the number of partitions in the cheap talk game depends on the misalignment signaled. Moreover, the ex ante expected utilities depend on the number of partitions, the misalignment signaled and the players types. Thus, since all these information are signaled in the first period, the game reduces to finding the players strategies played in the first period when signaling their types. The assumption that the players believe their signals as their true types completes this idea allowing to use the ex ante expected utilities defined in (22) and (23) as pay offs for the static game. □

At this point I summon the Crawford and Sobel (1982)'s results regarding the players' preferences over different kinds of equilibriums and variables that define the partition equilibrium. These are without proofs and can be found in the original manuscript.

Theorem 3 (Theorem 3 in Crawford and Sobel (1982))**.** *For given preferences (i.e., t^R, t^S), the receiver always strictly prefers equilibrium partitions with more steps (larger N's).*

Theorem 4 (Theorem 4 in Crawford and Sobel (1982))**.** *For given number of steps (i.e., N), the receiver always prefers equilibrium partition associated with more similar preferences (i.e. smaller difference $t^S - t^R$).*

Theorem 5 (Theorem 5 in Crawford and Sobel (1982))**.** *For given preferences (i.e., t^R, t^S), the sender always strictly prefers ex ante (before learning the information state realization) equilibrium partitions associated with more steps (larger N's).*

These results are a consequence of the monotonicity assumption proposed by Crawford and Sobel (1982). Theorems 3 and 4 hold even for ex post situations. The reason is based on the idea that more partitions involve more accurate messages. That means less distorted messages. More partitions are a consequence of more aligned players, since they share closer action preferences it is convenient for them to communicate better in such situations. However, theorem 5, only holds for ex ante situations. Given the sender's private knowledge of the information state, she might prefer even a smaller number of partitions. Some specific preferred understandings given an information state, where the sender utility is in expectation better than with any other partition, could be induced when communication with fewer partitions may occur. The difficulty of selecting this kind multiple equilibriums makes the model difficult to solve ex post.

The following Theorem characterizes the preferred strategies in a game, where the possible sender types in her distribution are all larger than the ones in the receiver distribution. I focus only in pure strategies equilibriums. For the existence of such, it is needed to assume that given the possible misalignments prior the game, there exists at least one that produces more partitions, and at least one that produces less than the others.

With a discrete distribution of types, each player chooses a signal constituted as one of the types in her distribution that maximizes her utility. As will be shown next, in this kind of games and under some distribution conditions, the signals constitute the misalignment with which the cheap talk equilibrium is to be constructed in the next game stages. The theorem shows that under some conditions there are dominant strategies. This equilibrium characterizes the conditions where only exists pooling equilibrium.

Theorem 6. *Assume the extended multistage signaling game of strategic type disclosure followed by a cheap talk game where the misalignment is determined in the first stage signaling game. There are two types distributed discretely for each player where $\underline{t}^R < \overline{t}^R < \underline{t}^S < \overline{t}^S$ and the smallest distance between sender and receiver types produces at least $N \geq 2$ number of partitions. Among the type differences there should be at least one that produces the smallest partition and at least one that produces the largest. Despite of her actual type, the sender is always strictly more interested in the equilibrium where the distance of types is the smallest for all possibilities in the distributions, and their associated prior and posterior probabilities are strictly larger than zero.*

Proof. It has to be shown the existence of a pooling equilibrium. It will suffice to determine which are the dominated strategies for different types. If such dominated strategies are the same despite the actual player type, then the pooling equilibria exists.

If $\underline{t}^R < \overline{t}^R < \underline{t}^S < \overline{t}^S$, the smallest difference of players types is $(\underline{t}^S, \overline{t}^R)$ and the largest is $(\overline{t}^S, \underline{t}^R)$. The number of partitions is denoted as $N(t^S, t^R)$, according Lemma 4 the higher the difference $t^S - t^R$ the lower the number of partitions.

For clarity's sake, I perform a change of notation for the rest of the proof. The types $\underline{t}^R < \overline{t}^R < \underline{t}^S < \overline{t}^S$ are renamed as $LR < HR < LS < HS$. Assume, $N(LS,HR) \geq N(HS,HR) \geq N(LS,LR) \geq N(HS,LR)$ implies $EU^R(N(LS,HR)) \geq EU^R(N(HS,HR)) \geq EU^R(N(LS,LR)) \geq EU^R(N(HS,LR))$.

The receiver prefers to signal the message HR that generates the largest number of partitions, and compares her expected utility against the one generated by signaling LR. In this case as postulated in (22), σ is the probability of the sender signaling her low type[9]. Notice that this calculation is the same despite any player type.

$$\sigma EU^R(N(LS,HR)) + (1-\sigma)EU^R(N(HS,HR)) > \sigma EU^R(N(LS,LR)) + (1-\sigma)EU^R(N(HS,LR)) \tag{24}$$

The receiver's strategy playing her higher type is preferred always when $EU^R(N(HS,HR)) > EU^R(N(LS,LR))$ for any value of $\sigma \in [0,1]$. When $EU^R(N(HS,HR)) = EU^R(N(LS,LR)) = EU^R(N(HS,LR))$ (in which case there must exist a $EU^R(N(HS,HR))$ larger than the all the others by the theorem assumption). Therefore, whenever the following linear combination $\sigma EU^R(N(LS,HR)) + (1-\sigma)EU^R(N(HS,HR))$ is larger than the highest possible value of the right hand side of the inequality, which happens when $\sigma > 0$. In this case the receiver plays her highest type whenever there is a positive probability (even really small but not strictly equal to zero) that the sender is playing her lowest type, and therefore reaching the most informative number of partitions. This allows the receiver to always pool her strategy independently her type. A similar analysis can be performed for the case where $N(LS,HR) \geq N(LS,LR) \geq N(HS,HR) \geq N(HS,LR)$. $\square$

For the sender a similar appreciation can be performed. The difference is that her expected utility depends on her true type. If the sender is far enough from the receiver's signal analyzed, this will hurt her utility function. However, the effect of signaling the type that is closer to the best response of the receiver is stronger than telling the truth. Then it is expected for the sender to pool her strategy while signaling her type.

6 Example

To illustrate the equilibrium derived in the previous section I present an example. The utilities of the sender and receiver are characterized as the leading example of Crawford and Sobel (1982), where the actions function are additive in its arguments satisfying the assumptions proposed.

[9] In the characterization of the expected utility (22) the multiplication of probabilities $q(\tilde{t}^S|t^S)\gamma(t^S)$ is equal to the probability of σ of sender signaling some of her types.

The information state is distributed uniformly on $[0,1]$ with cumulative distribution $F(m)$.

$$\begin{aligned} U^R &= -\left(a(y,t^R) - a(m,t^R)\right)^2 \\ &= -\left(y + t^R - (m + t^R)\right)^2 \\ &= -\left(y - m\right)^2 \end{aligned} \tag{25}$$

$$\begin{aligned} U^S &= -\left(a(y,t^R) - a(m,t^S)\right)^2 \\ &= -\left(y + t^R - (m + t^S)\right)^2 \\ &= -\left(y - m - (t^S - t^R)\right)^2 \end{aligned} \tag{26}$$

This utility representations are in line with Lemmas 1, 2 and 3. The difference of action preferences is present only in the sender's utility. This misalignment characterization is similar to the sender's bias approach that is common in the cheap talk literature. Since the distribution of the players types are all strictly larger for the sender, it can be guaranteed that the misalignment is always positive.

To put together the results of Theorem 6, it is require to find the equilibriums generated for the cheap talk game. As the characterization of the equilibrium is ex ante, the expected utilities of incurring in the game are to be calculated following this criteria. The best response of the receiver is based on the theoretical calculation defined in Definition 4. For some partition N and $i = 0,1,\ldots,N-1$

$$\bar{y}(a_i,a_{i+1}) = \frac{a_i + a_{i+1}}{2}. \tag{27}$$

The best response of the sender is characterized by the optimal partition of the message space given the misalignment. The calculations are to be performed disregarding the actual realizations of the this value. The sender solves the arbitrage condition (15) following the starting and finishing bin as postulated in Theorem 1, given the best response of the receiver of equation (27).

$$\begin{aligned} U^S\left(a\left(\frac{a_i + a_{i+1}}{2},t^R\right) - a(a_i,t^S)\right) &= U^S\left(a\left(\frac{a_{i-1} + a_i}{2},t^R\right) - a(a_i,t^S)\right) \\ -\left(\frac{a_i + a_{i+1}}{2} - a_i - (t^S - t^R)\right)^2 &= -\left(\frac{a_{i-1} + a_i}{2} - a_i - (t^S - t^R)\right)^2, \end{aligned} \tag{28}$$

solving for a_{i+1}

$$a_{i+1} = 2a_i - a_{i-1} + 4(t^S - t^R). \tag{29}$$

This second order difference equation can be iteratively solved for a_i, holding a_1 as a parameter and using the initial bin condition equate to zero as (17),

$$a_i = a_1 i + 2i(i-1)(t^S - t^R), \tag{30}$$

As it can be seen in the difference equation (30), the bin a_i is an additive combination of $a_1 i$ and $2i(i-1)(t^S - t^R)$. However, since the maximum bin is bounded by (18) to be equal to the unity at most, then the additive contribution of $2i(i-1)(t^S-t^R)$ cannot be larger than 1. Therefore, solving for the number of bins i (or number of partitions) given this last second order restriction $1>2i(i-1)(t^S-t^R)$ produces two solutions. From which, the solution that is needed is the one that that produces the maximum number of possible partitions. In order to find the maximum real number of bins given the solution, it is needed to refine this result with the use of the ceiling function, it can be found that the number of partitions is determined by the misalignment measure.

$$\left\lceil -\frac{1}{2}+\frac{1}{2}(1+\frac{2}{t^S-t^R})^{1/2}\right\rceil \tag{31}$$

The use of derivatives to observe the marginal contribution is risky in this case since the ceiling function is discontinuous. However, applying limits it can be found that when the t^S-t^R tends to infinity the optimal number of partitions is 1, on the other hand when the misalignment tends to zero, the number of partitions tends to infinity. This proposes the idea that while less conflict of interests exists between the players a more accurate message can be sent, since the sender has more partitions to choose from and make her message more specific.

This result preserves the findings of Crawford and Sobel (1982) referred to the fact that while the larger the attainable partition is (equal to the unity) the sender's message is non informative (he would signal 'the information state is in $[0,1]$'), since the receiver implies the same, as he would not even enter to the communication procedure, because he already knowns that the information state is between 0 and 1. It also shows that the length of the steps $[a_i,a_{i+1}]$ are growing while i is reaching N. This is referred to the fact that senders that observe a high realization of m cannot efficiently communicate it because of the misalignment existence. A receiver might believe that a high message can be due either to the fact of the senders type (t^S) or simply because of the information state is large. Before analyzing the players' strategies a last comment referred Crawford and Sobel (1982) findings is needed. The cut off number from which there is babbling equilibrium corresponds whenever values $t^S-t^R>1/4$ are in the game.

Following this best response, the ex ante expected utilities for the players as defined in (22) (23) can be found for any m and for all the possible messages associated with this. For that it is required to obtain some results derived from (30) present the solution.

If $i=N$ and $a_N=1$,

$$a_1=\frac{1-2N(N-1)(t^S-t^R)}{N}. \tag{32}$$

Therefore,

$$a_i=\frac{i}{N}+2(t^S-t^R)i(i-N) \tag{33}$$

and

$$a_i-a_{i-1}=\frac{1}{N}+2(t^S-t^R)(2i-N-1). \tag{34}$$

With this results, the ex ante expected utility for the receiver disregarding the type signals is characterized by

$$\begin{aligned}
EU^R &= \sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} -\Big[(-1)(m - \frac{a_{i-1}+a_i}{2})\Big]^2 dm \\
&= -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} \Big[(m - \frac{a_{i-1}+a_i}{2})\Big]^2 dm \\
&= -\frac{1}{12}\sum_{i=1}^{N}(a_i - a_{i-1})^3 \\
&= -\Big(\frac{1}{12}\sum_{i=1}^{N}\Big[\frac{1}{N} + 2(\tilde{t}^S - \tilde{t}^R)(2i - N - 1)\Big]^2\Big) \\
&= -\Big(\frac{1}{12N^2} + \frac{(\tilde{t}^S - \tilde{t}^R)^2(N^2-1)}{3}\Big),
\end{aligned} \tag{35}$$

and the ex ante expected utility for the sender disregarding the type signals and changing the notation for the misalignment equal to $Z = t^S - t^R$

$$\begin{aligned}
EU^S &= \sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} -\Big[(-1)(m + Z - \frac{a_{i-1}+a_i}{2})\Big]^2 dm \\
&= -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} \Big[(m + Z - \frac{a_{i-1}+a_i}{2})\Big]^2 dm \\
&= -\frac{1}{12}\sum_{i=1}^{N}(12Z^2 + (a_i - a_{i-1})^2)(a_i - a_{i-1}) \\
&= -\frac{1}{12}\sum_{i=1}^{N}(12Z^2 + (\frac{1}{N} + 2(\tilde{t}^S - \tilde{t}^R)(2i-N-1))^2)(\frac{1}{N} + 2(\tilde{t}^S - \tilde{t}^R)(2i-N-1)) \\
&= -\Big(\frac{1}{12N^2} + \frac{(\tilde{t}^S - \tilde{t}^R)^2(N^2-1)}{3} + Z^2\Big) \\
&= -\Big(\frac{1}{12N^2} + \frac{(\tilde{t}^S - \tilde{t}^R)^2(N^2-1)}{3} + (t^S - t^R)^2\Big).
\end{aligned} \tag{36}$$

It is worth to notice that the expected utility ex ante of both players are different. The sender's utility depends on the real misalignment that exists among the players. In contrast, the receiver is not interested in knowing for sure the type of the other player, but the signal that is going to generate the number of partitions $N(\tilde{t}^S, \tilde{t}^R)$.

When additionally taking the expectations for the type and the signal of the type, the expected utilities are defined as:

$$EU^R = -\sum_{T^S}\sum_{\tilde{T}^S}\Big(\frac{1}{12N^2} + \frac{(\tilde{t}^S - \tilde{t}^R)^2(N^2-1)}{3}\Big) q(\tilde{t}^S|t^S)\gamma(t^S) \tag{37}$$

$$EU^S = -\sum_{T^R}\sum_{\tilde{T}^R}\Big(\frac{1}{12N^2} + \frac{(\tilde{t}^S - \tilde{t}^R)^2(N^2-1)}{3} + (t^S - t^R)^2\Big) q(\tilde{t}^R|t^R)\delta(t^R) \tag{38}$$

With these last two equations, pay offs for both players can be constructed when they are in period 1.

As part of the example I propose the following types distribution that satisfies $\underline{t}^R < \overline{t}^R < \underline{t}^S < \overline{t}^S$. The proposed model requires that these are common knowledge among the players,

$$t^R = \begin{cases} 0.3 & \text{with probability } \delta \in [0,1] \\ 0.2 & \text{with probability } 1-\delta \end{cases} \tag{39}$$

$$t^S = \begin{cases} 0.5 & \text{with probability } \gamma \in [0,1] \\ 0.47 & \text{with probability } 1-\gamma \end{cases} \tag{40}$$

The associated misalignment for the combination of types satisfy Theorem 6 requirements. Moreover, there is at least one that produces the highest number of partitions. In this example $N(0.5, 0.3) = 2$, $N(0.47, 0.3) = 2$, $N(0.5, 0.2) = 1$ and $N(0.47, 0.2) = 1$. Theorem 6 suggests that there is a receiver's strategy that dominates the other despite the player's type. Therefore, the receiver should always signal that her type is 0.3. Solving for every player strategy and type, the pay offs are presented in the following tables, as normal representations.

	$t^S = 0.5$, $t^R = 0.3$			
	$\tilde{t}^S = 0.5$		$\tilde{t}^S = 0.47$	
$\tilde{t}^R = 0.3$	-0.06083	-0.10083	-0.04973	-0.08973
$\tilde{t}^R = 0.2$	-0.08333	-0.12333	-0.08333	-0.12333

	$t^S = 0.5$, $t^R = 0.2$			
	$\tilde{t}^S = 0.5$		$\tilde{t}^S = 0.47$	
$\tilde{t}^R = 0.3$	-0.06083	-0.15083	-0.04973	-0.13973
$\tilde{t}^R = 0.2$	-0.08333	-0.17333	-0.08333	-0.17333

	$t^S = 0.47$, $t^R = 0.3$			
	$\tilde{t}^S = 0.5$		$\tilde{t}^S = 0.47$	
$\tilde{t}^R = 0.3$	-0.06083	-0.08973	-0.04973	-0.07863
$\tilde{t}^R = 0.2$	-0.08333	-0.11223	-0.08333	-0.11223

	$t^S = 0.5$, $t^R = 0.2$			
	$\tilde{t}^S = 0.5$		$\tilde{t}^S = 0.47$	
$\tilde{t}^R = 0.3$	-0.06083	-0.13373	-0.04973	-0.12263
$\tilde{t}^R = 0.2$	-0.08333	-0.15623	-0.08333	-0.15623

With equations (37) and (38), the results of Theorem 6 can be depicted. As announced before, the receiver has the same pay offs in all the tables, which means that she is only affected by the type signals but not by the types themselves. From revising the Bayesian Nash equilibrium, it is concluded that she has a pure strategy of always playing her higher type. Solving equation (37) when the receiver is playing 0.3 against the strategy of signaling 0.2, it can be seen that regarding any probability between $[0, 1]$, she is going to play her preferred strategy that provides her the Nash equilibrium.

For the sender's case, her pay offs depends on hers and the receiver's type. Solving equation (38) and maximizing it for her best strategy, it can be seen that she is going to prefer to play her lower signal no matter what type she really is. However there is a condition when solving the inequalities for the sender when comparing her strategies. She is going to always signal her lower type whenever the probability of the receiver signaling her higher type is going to be strictly larger than zero or in the range $(0, 1]$. That means that she is always going to pool whenever (no matter with which low probability but not zero) the receiver is signaling her higher type. On the other case, with probability zero, there should be a hybrid equilibrium.

As it has been shown, both players pool their strategies searching always for meaningful communication to occur. This exercise can be performed with different players types[10], and even with very large differences between their types, whenever the conditions of the Theorem 6 are met, there exists the opportunity to incur in meaningful[11] communication.

The model presents an equilibrium strategy whenever the ex ante condition is met. However, when analyzing the ex post case, it can be seen that the quality of communication is poor, and improves only the welfare of the sender in comparison to Crawford and Sobel (1982) model. Moreover, in situations ex post, no equilibrium exists, since the receiver knows that it is only in the benefit of the sender, who is able to signal more accurate messages in the direction of her preferences (which in certain situations can be very misaligned), leading to several loss of welfare in the case of the receiver, who would perform actions closer to the sender preferred actions. Then, in ex post situation, there is no equilibrium.

7 Conclusions

In this study, I show that whenever there is a period before engaging communication via cheap talk to signal each others preferences, exists a pooling equilibrium regarding their strategies. This pooling equilibrium shows the willingness of both players to signal themselves as closer to each others type no matter their true nature. The intuition of this results is referred to the fact that both players are highly interested in informational communication and are willing to lie in order to let communication to occur even in situations where the basic model showed that this kind of communication was not possible. The theoretical approach holds for the receiver's strategy whenever the players have quadratic loss utility functions with additive action functions. The sender's strategy is shown to hold empirically as well. This class of study is performed whenever the sender may present a larger preference than the receiver's for all available types.

[10]For example, it can be consider the case where the receiver's types are $0.01, 0.1$ and the sender's $0.3, 0.99$, with similar results. The difference is that in the latter, the receiver is always going to pool, whenever the sender's probability of playing 0.3 is strictly larger than zero as predicted by Theorem 6.

[11]Meaningful in the sense that it provides a better equilibrium than babbling equilibrium.

Ex ante examination of the expected utilities is a key characteristic of the analysis to ensure the possibility of communication. The equilibrium selection analysis is not examined, since the main focus is to show the players behavior while playing cheap talk. Naturally and first of all, a strategic player would lie about her types (bias). Allowing a possibility of informational communication that is not going to happen, especially after the moment that the sender observes the information state, and takes the advantage of the informational communication window to use convincing messages to persuade the receiver to perform the sender's preferred action.

Although the model in ex post situations was not formally studied, the example conducted suggests a collapse of the equilibrium in such situation. Which means, that what it is appealing ex ante, ex post may be useless. The main problem in this kind of games in ex post situation is the way how the players update their beliefs about m given the updated belief of t and the signal n, moreover, the conditional probability of n given the updated belief of the type. The reason why with this model, ex post analysis is not possible traces to the assumption about the players commitment to take as true the type signals emitted in the first period. However, without it the ex ante communication would not be feasible because of the updating beliefs problems.

I contemplate several ways to conduct new approaches about this topic. It seems that in order to open the possibility of an ex post analysis in equilibrium, it is required to allow noisier communication to happen. Verifiability in cheap talk models have been already been developed as noticed in the literature review, and the application of such characteristic might add enrichment to the characterization of this kind of game. The solution possibly can be worked as cheap talk with several biased receivers and several biased senders. Another avenue seems to be the use of a supermodularity approach, since it does not rely in bayesian updating.

Nevertheless, this model shares an agreement in its results together with others in the literature of bias uncertainty, specifically with Li and Madarász (2008), about the fact that under the uncertainty of the players misalignment, informational communication happens more often but its meaning is diminished.

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