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### **Liquidity Shocks and the Dollarization of a Banking System**

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# Liquidity Shocks and the Dollarization of a Banking System\*

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## Abstract

This paper shows how uncertainty about liquidity demand can lead to a high degree of dollarization in the banking system. I study a model where the demand for currency in each period is random, and where it is easier for banks to borrow in local currency in times of crisis than in dollars. Banks choose a portfolio composed of local currency, dollars, and real loans. Compared to the anticipated transactions demand for each currency, I show that the bank will hold a relatively large amount of dollars and a relatively small amount of local currency. I also show the existence of a *dollarization multiplier*: as the anticipated transactions demand for dollars increases, the dollarization of the banking sector increases more than proportionately.

*Keywords:* Dollarization, Banking crisis, Banking System

*JEL classification:* F31, G21, G33.

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# 1 Introduction

Much of the recent literature on dollarization has focused on the characteristics of partially dollarized economies, where dollars and local currency each account for a substantial fraction of economic transactions. This literature has identified the degree of dollarization of the banking system as being a particularly important variable.<sup>1</sup> The effects of monetary policy, for example, as well as the reaction of the economy to external shocks, appear to depend critically on the degree to which domestic banks choose to denominate their transactions in dollars. This raises the important question of what determines the level of dollarization in the banking system.

Most of the existing explanations focus on "liability dollarization," in which depositors choose to have their deposits denominated in dollars. The literature is therefore concerned largely with explaining why people would choose to open dollar-denominated accounts (see Broda and Levy Yeyati [4], Calvo [5], Catão and Terrones [7], Savastano [19] among others).<sup>2</sup> Other explanations have concentrated on network externalities that appear when banks' transactions are made in foreign currency. A lot of work in this area has been done with regard to Bolivia, which is one of the main examples of a partially dollarized economy (see for example Peiers and Wrase [17], Reding and Morales [18] and Cuddington, *et al.*[9]).<sup>3</sup> This work is closely related to the well known phenomenon of "hysteresis," which was first introduced to the study of dollarization issues by Uribe [23].

In this paper, I concentrate on the other side of a bank's balance sheet. I show that uncertainty about liquidity demand will also tend to push a banking system toward becoming highly dollarized. The model I develop is a generalization of Champ, Smith, and Williamson [8].<sup>4</sup> I transform their one-currency model into a two-currency model, introducing the possibility for banks to hold reserves in either local or foreign currency. Hereafter I will refer to these currencies as pesos and dollars, respectively. In each period, the demand for currency is stochastic. A fraction of the agents who demand currency will need pesos, and the remaining fraction will need dollars.

I make the natural assumption that in times of high liquidity demand, it is easier for a bank to borrow in local currency than in dollars. I show that this assumption implies that, given some anticipated transactions demand for each currency, banks will choose to hold a relatively large amount of dollars reserves and a relatively small amount of reserves in local currency. Uncertainty about liquidity demand therefore leads to a form of "asset dollarization" in addition

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<sup>1</sup>See, for instance, De Nicoló, *et al.*[10] who study the benefits and risks associated with a dollarized banking system and Castro, *et al.* [6] who analyze the financial vulnerability of partially dollarized economies.

<sup>2</sup>For a complete review of the literature see Levy Yeyati and Sturzenegger [15].

<sup>3</sup>The Bolivian banking system is highly dollarized, 80% of total assets are in dollars and liability dollarization is around 90%.

<sup>4</sup>Antinolfi, Huybens and Keister [2], Bhattacharya, Haslag and Rusell [3], Haslag and Martin [13], Schreft and Smith [20] and Smith [21] also are interesting generalizations, but for the one-currency model.

to the liability dollarization studied in other papers.

During a liquidity crisis, a bank will be able to meet the demand for pesos, since it can borrow pesos from the Central Bank. However this will not be true for dollar demand. This asymmetry leads a bank to hold a "precautionary" stock of dollars and the model will show how this is done. In fact it will be shown that the demand for dollar reserves is in large part driven by "rare events" or "extreme events". I concentrate first on the worst-case scenario, where there is no possibility for banks to borrow dollars from the Central Bank. I then generalize the results by introducing the possibility of borrowing in dollars at a positive interest rate.

The main idea of the model, that banks are self-insuring, is not exclusive to a dollarized economy or banking system. For example, Antinolfi and Keister [1] show, in a single currency model, how different Central Bank policies affect the equilibrium levels of cash reserves and of real investment held by banks. However, the question of how banks choose to dollarize their assets has not been modeled yet in a clear and useful manner, and this paper aims to fill that gap.

The remainder of the paper proceeds as follows. In section 2, the two-currency model of money and liquidity demand is developed. Section 3 studies the equilibria of the model when banks cannot borrow any dollars from the Central Bank. The "dollarization multiplier" is defined and derived. In section 4, I generalize the results to a setting where it is possible for banks to borrow some dollars from the Central Bank. Finally, in section 5, I present some concluding remarks.

## 2 The Basic Model

In this section I describe the main elements of the model, based on Champ, Smith and Williamson [8] and modified in order to address the issue of a partially dollarized banking system.

### 2.1 The Environment

This is an overlapping generations model (OLG), where agents live for two periods and there is an initial old generation. There is a single, perishable consumption good. At each date  $t = 0, 1, \dots$ , a continuum of agents with unit mass is born at each of two identical locations (islands). Half of these agents are "lenders" who have endowments  $(\omega_1, \omega_2) = (x, 0)$ , and the other half are borrowers, who have an endowment vector given by  $(\omega_1, \omega_2) = (0, y)$ .<sup>5</sup> The consumption set for all consumers is  $\mathbb{R}_{++}^2$  and their utility function is given by  $u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$ . The first assumption made is that  $\beta x > y$  holds, which implies that this is a "Samuelson case" economy (see Gale [12]) and hence there is a role for money as a store of value. The distinguishing characteristic of

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<sup>5</sup>The fraction of the population in each group is not important; one half is chosen arbitrarily. All that matters is the total endowment of each group.

this economy is that there are two currencies in circulation, pesos and dollars, the former of which is issued by the Central Bank. Agents in the initial old generation have  $M > 0$  units of fiat money, all denominated in pesos, the stock of pesos is assumed to be constant over time.

Both currencies share the role of store of value, but they also share a transactions role which is generated by the random relocation of people between the two spatially separated islands (see Townsend [22]). This setup allows money to be dominated in rate of return by other assets. In addition to the two domestic islands, there is an "outside" or "foreign" island that represents the rest of the world. A fraction  $\pi_t$  of young lenders in each of the domestic islands is notified that they will be moved to either the other domestic island or the foreign one. From the fraction  $\pi_t$  of young relocated lenders, a fraction  $\theta$  will move to the domestic island and will need pesos, and a fraction  $1 - \theta$  will move to the foreign island and will need dollars. In this way a demand for both dollars and domestic currency is generated. Here, I assume  $\theta > \frac{y}{\beta x}$ ; as will become clear later, this assumption implies that the transaction demand for dollars is never "too large".

The timing of events is as follows: At the beginning of each period  $t$ , a new generation of agents is born. Agents receive their endowments and young lenders can deposit them in a bank or can trade with the old agents. At this point, agents cannot move between or communicate across locations. Goods can never be transported between domestic locations. Hence, goods and asset transactions occur autarkically within each location. Also, young borrowers can get loans from the bank. There are also risk-sharing issues, because young lenders do not know whether or not they will be movers (see Diamond and Dybvig [11]). After deposits have been made, banks have the opportunity to buy dollars on the international market using goods they have received from depositors. This market meets only once, before relocation shocks are realized. After this opportunity has passed, banks are unable to acquire more dollars until the next period.<sup>6</sup> Relocation shocks then are realized, and as people know where they are going to move, they withdraw either pesos or dollars from their bank. At the end of period  $t$ , relocation actually occurs.

At time  $t + 1$ , agents receive their old-age endowments, and borrowers use part of this endowment to repay their loans. With this revenue, banks make repayments to lenders who did not move. Old agents can trade with young lenders and/or banks. At this point, all old agents consume and then die. Note that the old-age consumption of a mover will always be equal to the real value of the money that she takes with her to the new location.

The relocation probability  $\pi_t$  is a random variable in each period that gives the size of the aggregate liquidity shock; high values of  $\pi_t$  correspond to high liquidity demand. It has support  $[0, 1)$  and is drawn from the twice continuously differentiable, strictly increasing distribution function  $F$  with associated density

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<sup>6</sup>The timing of events prevents an agent who is located domestically from carrying dollars to the other location and using them to buy consumption there. The assumptions capture the idea that dollars are not perfect substitutes for domestic currency; some transactions must take place in pesos. Alternatively, one could simply assume that domestic movers must use pesos due to legal restrictions on the use of foreign currency.

function  $f$ . It is independently and identically distributed over time. A higher  $\pi_t$  implies a higher demand for both currencies.

Loans and both types of money are seen as assets for the bank, so the bank will divide its portfolio between loans, dollars, and pesos, depending on the return that each asset offers.

The price level  $p_t$  is determined in the market that meets at the beginning of the period. In this market, there is no randomness since the bank does not yet know the realization of  $\pi_t$ . After  $\pi_t$  is realized, there are no markets open until the next period. In a way, this means that liquidity demand is stochastic in the model, but money demand is not. I also assume that the economy is small enough that it takes the price of dollars as given by the world market. This price is normalized to  $p_t^* = 1$ .

## 2.2 Borrowers

Borrowers, who never move, face a gross real interest rate of  $R_t$ . Let  $l_t$  be the size of the loan demanded. Then they choose the quantity they wish to borrow,  $l_t$ , by solving the problem

$$\max_{l_t} \ln(l_t) + \beta \ln(y - R_t l_t)$$

The solution to this problem is given by

$$l_t = \frac{y}{(1 + \beta)R_t} \quad (1)$$

## 2.3 Lenders

Lenders face a more complicated problem. Given that they are faced with random relocation, they deposit all of their savings in a bank and receive a return that depends on whether or not they move and, if they move, whether they are movers in dollars or movers in pesos. Specifically, they are promised a real return  $r_t(\pi)$  if they do not move,  $r_t^m(\pi)$  if they move and demand pesos, and  $r_t^*(\pi)$  if they move and demand dollars. Lenders then choose the amount they save and deposit  $d_t$  to maximize expected utility, that is, to solve

$$\max \ln(x - d_t) + \beta \int_0^1 \{ \pi [\theta \ln(r_t^m(\pi) d_t) + (1 - \theta) \ln(r_t^*(\pi) d_t)] + (1 - \pi) \ln(r_t(\pi) d_t) \} f(\pi) d\pi$$

The solution to this problem sets

$$d_t = \frac{\beta x}{(1 + \beta)} \quad (2)$$

It has to be noted that the income and substitution effects of a change in the rate of return exactly offset each other; this fact clearly depends on the assumption of log utility and no old-age income for lenders. As in Champ *et al.* [8] and others, this assumption allows the model to be solved analytically.

## 2.4 Banks

Banks take deposits, make loans, hold reserves, announce return schedules and can borrow pesos from the Central Bank at a zero interest rate (if needed). I assume that banks can only borrow in local currency. This assumption is somewhat strong, in reality banks can usually borrow some amount of dollars. However, I show below that relaxing this assumption and allowing banks to borrow dollars at a positive nominal interest rate does not change the results.<sup>7</sup>

Borrowers can write loan contracts with any bank and banks behave competitively in the sense that they take the real return on assets as given. On the deposit side, banks are assumed to behave as Nash competitors, which leads them to choose state-contingent deposit returns to maximize the expected utility of young lenders. Let's introduce some notation about the variables that will be important for the bank:

**Notation 1**  $z_t$  = real value of cash reserves per depositor in pesos;  $z_t^*$  = real value of cash reserves per depositor in dollars;  $d_t - z_t$  = real value of loans made per depositor in pesos;  $d_t - z_t^*$  = real value of loans made per depositor in dollars;  $\gamma_t = \frac{z_t}{d_t} \rightarrow$  reserve-deposit in pesos;  $\gamma_t^* = \frac{z_t^*}{d_t} \rightarrow$  reserve-deposit in dollars;  $\delta_t = \frac{b_t}{d_t} \rightarrow$  real borrowing per unit of deposits in pesos;  $\delta_t^* = \frac{b_t^*}{d_t} \rightarrow$  real borrowing per unit of deposits in dollars.

The bank faces three constraints on the return schedules it can offer. Relocated agents must be given pesos or dollars, since this is the only asset which will allow these agents to consume in their new locations. These currencies come from the bank's reserve holdings and from borrowing from the Central Bank. Let  $\alpha_t(\pi)$  denote the fraction of the bank's peso reserves that is given to movers and  $\alpha_t^*(\pi)$  the fraction of the bank's dollar reserves that is given to movers. As  $p_t$  is the general price level at time  $t$ ,  $\frac{p_{t+1}}{p_t}$  is the return to holding pesos between time  $t$  and  $t+1$ . The return on holding dollars is, by assumption, one. The next two equations are the constraints for movers in pesos and in dollars respectively and the third one is the constraint for non-movers.

$$\pi \theta r_t^m(\pi) \leq \alpha_t(\pi) \gamma_t \frac{p_{t+1}}{p_t} + \delta_t(\pi) \frac{p_{t+1}}{p_t} \quad (3)$$

$$\pi(1 - \theta) r_t^*(\pi) \leq \alpha_t^*(\pi) \gamma_t^* \quad (4)$$

$$(1 - \pi) r_t(\pi) \leq (1 - \alpha_t(\pi)) \gamma_t \frac{p_{t+1}}{p_t} + (1 - \alpha_t^*(\pi)) \gamma_t^* + (1 - \gamma_t - \gamma_t^*) R_t - \delta_t(\pi) \frac{p_{t+1}}{p_t} \quad (5)$$

The left hand sides of equations (3) and (4) represent the total real value of goods that will be given to movers in pesos and in dollars respectively. Equation

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<sup>7</sup>See Morales [16] for a discussion of a Central Bank charging punitive interest rates on its lender-of-last-resort loans.

(5) is the payment to non-movers, which cannot exceed the value of the bank's remaining portfolio of reserves plus the returns from the bank's lending minus the repayment of the Central Bank's loan. Note that the bank's loans earn the real rate of return  $R_t$ .

Substituting these constraints into the bank's objective function we have to maximize:

$$\max \int_0^1 \left\{ \begin{aligned} & \pi \theta \ln(\alpha_t(\pi) \gamma_t + \delta_t(\pi)) + \\ & \pi(1-\theta) \ln(\alpha_t^*(\pi) \gamma_t^*) + \\ & (1-\pi) \ln \left[ \frac{(1-\alpha_t(\pi)) \gamma_t \frac{p_{t+1}}{p_t} + (1-\alpha_t^*(\pi)) \gamma_t^* +}{(1-\gamma_t - \gamma_t^*) R_t - \delta_t(\pi) \frac{p_{t+1}}{p_t}} \right] \end{aligned} \right\} f(\pi) d\pi$$

subject to  $\begin{array}{lll} 0 \leq \alpha_t(\pi) \leq 1 & 0 \leq \gamma_t \leq 1 & 0 \leq \delta_t(\pi) \\ 0 \leq \alpha_t^*(\pi) \leq 1 & 0 \leq \gamma_t^* \leq 1 & \end{array}$

Notice that (i) the fraction of peso reserves paid to movers  $\alpha_t$ , (ii) the fraction of dollar reserves paid to movers  $\alpha_t^*$ , and (iii) the real amount of borrowing in pesos  $\delta_t$ , are all chosen after the realization of  $\pi$ . On the other hand,  $\gamma_t$  and  $\gamma_t^*$ , the fraction of reserves in the bank's asset portfolio, is chosen before the realization of  $\pi$ .

First, one can note that when the realized value of  $\pi$  is small, banks will give only a fraction of their cash reserves to movers and the inequality constraints in the problem will not be binding. In this case the solution sets:

$$\alpha_t(\pi) = \pi \theta \left[ 1 + \frac{\gamma_t^*}{\gamma_t} \frac{p_t}{p_{t+1}} + \frac{R_t}{\gamma_t} \frac{p_t}{p_{t+1}} (1 - \gamma_t - \gamma_t^*) \right] - \frac{\delta_t(\pi)}{\gamma_t} \quad (6)$$

and

$$\alpha_t^*(\pi) = \pi(1-\theta) \left[ 1 + \frac{\gamma_t}{\gamma_t^*} \frac{p_{t+1}}{p_t} + \frac{R_t}{\gamma_t^*} (1 - \gamma_t - \gamma_t^*) \right] \quad (7)$$

This solution is valid as long as the inequality constraints are indeed satisfied, which is true if  $\pi$  is no greater than

$$\pi^* = \frac{1}{\theta} \left[ \frac{\gamma_t \frac{p_{t+1}}{p_t}}{\gamma_t \frac{p_{t+1}}{p_t} + \gamma_t^* + R_t(1 - \gamma_t - \gamma_t^*)} \right] \quad (8)$$

Then, for a low value of  $\pi$ , specifically for values of  $\pi \in [0, \pi^*]$ , the values of  $(\alpha, \alpha^*)$  are the solution, and it can be easily checked for these values:

$$r_t^m(\pi) = r_t^*(\pi) = r_t(\pi) = \gamma_t \frac{p_{t+1}}{p_t} + \gamma_t^* + R_t(1 - \gamma_t - \gamma_t^*) \quad (9)$$

This means that when the demand for liquidity is low (the relocation shock is below a critical value  $\pi^*$ ), the bank is able to give movers of both currencies and non-movers the same return by paying out only a fraction of its reserves to movers. Since the bank wants to provide lenders with insurance against the relocation shock, this is the optimal thing to do.



Next, if  $\pi$  is greater than  $\pi^*$ , we know that the  $\alpha = 1$  constraint will be binding. That is, the bank will be giving all of its peso reserves to movers, and may need to borrow additional pesos from the Central Bank, so  $\delta > 0$ . In this case, the solution to the problem is:

$$\delta_t(\pi) = \pi\theta \left[ \gamma_t + \gamma_t^* \frac{p_t}{p_{t+1}} + R_t(1 - \gamma_t - \gamma_t^*) \frac{p_t}{p_{t+1}} \right] - \gamma_t \quad (10)$$

$$\alpha_t^*(\pi) = \pi(1 - \theta) \left[ 1 + \frac{\gamma_t}{\gamma_t^*} \frac{p_{t+1}}{p_t} + \frac{R_t}{\gamma_t^*} (1 - \gamma_t - \gamma_t^*) \right] \quad (11)$$

What is interesting here, and it is straightforward to prove, is that even for values of  $\pi$  greater than  $\pi^*$ , the bank will again choose to guarantee the same return to everyone, which is equal to the average return on the bank's portfolio:

$$r_t^m(\pi) = r_t^*(\pi) = r_t(\pi) = \gamma_t \frac{p_{t+1}}{p_t} + \gamma_t^* + R_t(1 - \gamma_t - \gamma_t^*) \quad (12)$$

In other words, for larger values of  $\pi$  (the relocation shock is beyond the critical value  $\pi^*$ ) the bank is also able to provide lenders with insurance against the relocation shock, by offering them the same return. How could this be possible? The answer is that the bank can always borrow in pesos, so if people come to the bank and ask for pesos, the bank will satisfy everybody. This is intuitive since we can think that the Central Bank is always able to print local currency to satisfy the demand of the movers in pesos. Things become difficult for the bank when the proportion of movers in dollars who come to the bank is large.

Finally banks will then set  $\gamma_t$  and  $\gamma_t^*$  to maximize the return on the bank's portfolio which is given by the right hand of equation (9) or (12). The optimal choice of reserve-deposit ratio in pesos  $\gamma_t$  must therefore be given by:

$$\gamma_t = \left\{ \begin{array}{c} 0 \\ \in [0, 1] \\ 1 \end{array} \right\} \text{ as } \frac{p_{t+1}}{p_t} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} R_t$$

and for the optimal reserve-deposit ratio in dollars, we have to compare the return on dollars which is equal to 1 with the return on loans  $R_t$ :

$$\gamma_t^* = \left\{ \begin{array}{c} 0 \\ \in [0, 1] \\ 1 \end{array} \right\} \text{ as } 1 \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} R_t$$

So at the time a bank has to decide in which assets to invest, it will compare the returns of the three possibilities (pesos, loans and dollars). What if  $\frac{p_{t+1}}{p_t} = R_t < 1$ ? Recall that  $\gamma_t + \gamma_t^* \leq 1$  must hold always, so if this is the case  $\gamma_t^*$  will be equal to one and  $\gamma_t$  will be equal to zero, so the bank will prefer to hold all of its assets in dollars.

### 3 Equilibrium Conditions

An equilibrium of this economy is characterized by the market clearing conditions for real balances in local currency and loans. Because the supply of real balances is equal to  $\frac{M}{p_t}$  and the demand for real balances in pesos is given by  $\gamma_t d_t$ , market clearing for real balances and (2) require:

$$\gamma_t d_t = \gamma_t \frac{\beta x}{1 + \beta} = p_t M \quad (13)$$

Similarly, the demand for loans is given in (1), while the supply of loans is given by  $(1 - \gamma_t - \gamma_t^*) d_t$ . Together these yield the market clearing condition for loans,

$$\frac{y}{(1 + \beta) R_t} = (d_t - z_t - z_t^*) = (1 - \gamma_t - \gamma_t^*) d_t = (1 - \gamma_t - \gamma_t^*) \frac{\beta x}{1 + \beta} \quad (14)$$

To solve for the equilibrium we are going to use the assumption about  $\theta$  made before, which implies that a large enough proportion of movers will demand pesos. This, and defining a critical value of  $\theta$  as  $\bar{\theta} \equiv \frac{y}{\beta x}$ , gives us the following proposition:

**Proposition 1** *If  $\theta > \bar{\theta} \equiv \frac{y}{\beta x}$ , a stationary equilibrium exists, and  $\gamma_t + \gamma_t^* = 1 - \frac{y}{\beta x}$  and  $\gamma^* \geq 1 - \theta$ .*

**Proof.** From the equilibrium condition (14), we have that  $(1 - \gamma_t - \gamma_t^*) R_t = \frac{y}{\beta x}$ . In equilibrium we know that  $R_t = 1$ , then  $(1 - \gamma_t - \gamma_t^*) = \frac{y}{\beta x}$ . From equation (13) we know that  $\gamma_t$  could never be equal to zero, since this would imply that  $p_t = 0$ , this would mean that pesos have no value, and nobody would have the incentive to hold pesos at all. So we have to look for a solution where both  $\gamma_t$  and  $\gamma_t^*$  satisfy the equilibrium conditions and it is straightforward to see that the solutions proposed for  $\gamma$  and  $\gamma^*$  satisfy them. In order to prove that this is an equilibrium we need to verify also that  $\alpha_t^*(\pi)$  in (11) is less than (or equal to) one for all possible values of  $\pi$ . In fact using the equilibrium conditions, it can be shown that  $\alpha_t^*(\pi = 1) \leq 1$  if and only if  $\gamma^* \geq 1 - \theta$ .<sup>8</sup> (Q.E.D.)

Therefore, what I have proved is that when it is assumed that banks cannot borrow any dollars from the Central Bank, we have a continuum of equilibria that satisfy:

$$\gamma_t + \gamma_t^* = 1 - \frac{y}{\beta x} \quad (15)$$

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<sup>8</sup>Note that  $\alpha_t^*(\pi = 1) = \frac{1 - \theta}{\gamma_t^*}$ .

It remains to show how the bank will make its choice between  $\gamma$  and  $\gamma^*$  or how many reserves it will hold in pesos and in dollars. As stated above, the bank will hold enough dollars to cover the worst possible case, as it knows that the Central Bank will not lend them any dollars in any case. In this way, when  $\theta > \bar{\theta}$  the bank will set  $\gamma^* \geq 1 - \theta$  and  $0 \leq \gamma \leq \theta - \frac{y}{\beta x}$ .

This is a case of continuum of steady state equilibria, which is a common result in OLG models with two currencies. This result was shown first by Kareken and Wallace [14] where, in equilibrium, both currencies are perfect substitutes in some sense. They also showed exchange rate indeterminacy, something also present in my model: in the different stationary equilibria, the real value of a peso is different and therefore the exchange rate is different.

I focus on the "minimal dollarization" equilibrium, where  $\gamma^* = 1 - \theta$ . The next proposition shows that, even in this equilibrium, the equilibrium level of dollarization in the banking system is large relative to the transactions demand for dollars. All other (stationary) equilibria involve the bank holding an even larger quantity of dollars and, therefore, the results holds in an even stronger sense there. In the minimal dollarization equilibrium, we have  $\gamma = \theta - \frac{y}{\beta x}$  and we can define  $\Psi$  as the "level of dollarization", that is, as the ratio of the real value of dollar reserves to the real value of all cash reserves.

$$\Psi = \frac{\gamma^*}{\gamma + \gamma^*} \quad (16)$$

The following proposition states the main result of the paper:

**Proposition 2** *As the demand of dollars increases, the dollarization of the banking system increases more than proportionately, i.e.  $\left| \frac{\partial \Psi}{\partial \theta} \right| > 1$ .*

**Proof.** Substituting the minimal dollarization equations for  $\gamma$  and  $\gamma^*$  in equation (16), we get  $\Psi = \frac{1-\theta}{1-\frac{y}{\beta x}}$ . It is easy to see that  $\frac{\partial \Psi}{\partial \theta} = -\frac{1}{1-\frac{y}{\beta x}}$ , so  $\left| \frac{\partial \Psi}{\partial \theta} \right| > 1$ . (Q.E.D.)

I call  $\frac{\partial \Psi}{\partial \theta}$  the "dollarization multiplier". The fact that this multiplier is larger than one shows clearly that the dollarization of the banking system grows faster than the transaction demand for dollars. In other words, as  $\theta \rightarrow \bar{\theta} \implies \gamma \rightarrow 0$  and so  $\gamma^* \rightarrow 1 - \frac{y}{\beta x}$ . In terms of the multiplier, this means that  $\Psi \rightarrow 1$  as  $\theta \rightarrow \bar{\theta}$ . This is a new explanation for a dollarized banking system, based on liquidity shocks and precautionary demand.

Let us see what the model says about the relation between asset and liabilities in dollars. Is there something that can explain why, in terms of levels, banks might have more assets in dollars than liabilities? Although it is not the primary aim of the model, the following corollary says something about it.

**Corollary 1** *In a stationary equilibrium, banks will hold more assets in dollars than liabilities.*

**Proof.** Equation (12) shows that in a stationary equilibrium, real return parity holds. Since  $R_t = 1$  and  $p_{t+1} = p_t$ , it can be seen that all movers receive

a return of one, regardless of whether they need pesos or dollars and regardless of the realization of  $\pi$ . We can call this return the "equilibrium return on early withdrawals". In this way dollar liabilities will be equal to  $(1 - \theta) \times 1$ . Then substituting for  $\gamma$  and  $\gamma^*$  in the dollarization multiplier —equation (16)— and using the assumption about the Samuelson case economy, it can be seen that:  $\frac{1-\theta}{1-\frac{\theta}{\beta_x}} > 1 - \theta$ . (Q.E.D)

The definition of liabilities given above is analogous to the way that the liabilities of a real bank would be defined: What has the bank "promised" to people if they all demand to withdraw early? Of course, not everyone will attempt to withdraw early; some people will be non-movers and will wait until the next period to withdraw.

It remains to talk about what happens when  $\theta$  is below  $\bar{\theta}$ . In this situation  $\gamma^* < 1 - \theta$ , thus banks will occasionally run out of dollars. This means the solution to the bank's problem will now be different; the bank will want to hold even more dollars (for precautionary purposes). But then there is no demand for reserves in local currency, and therefore pesos will have no value. Restoring the existence of equilibrium in this case would require modifying the model in some way in order to generate a lower bound for the demand for pesos. Such modifications are beyond the scope of the present paper and are left for future research.

## 4 Generalizing the results

Finally, I am going to relax the assumption about not borrowing dollars from the Central Bank and instead suppose that banks can indeed borrow dollars, but at a "penalty rate".<sup>9</sup> By a "penalty rate," I mean that the interest rate on borrowing in dollars is higher than the market rate. As Morales [16] says, it is an interest rate that makes banks believe that Central Bank liquidity is expensive.<sup>10</sup>

In this case the bank's constraints will be given by:

$$\pi \theta r_t^m(\pi) \leq \alpha_t(\pi) \gamma_t \frac{p_{t+1}}{p_t} + \delta_t(\pi) \frac{p_{t+1}}{p_t} \quad (17)$$

$$\pi(1 - \theta) r_t^*(\pi) \leq \alpha_t^*(\pi) \gamma_t^* + \delta_t^*(\pi) \quad (18)$$

$$\begin{aligned} (1 - \pi) r_t(\pi) \leq & (1 - \alpha_t(\pi)) \gamma_t \frac{p_{t+1}}{p_t} + (1 - \alpha_t^*(\pi)) \gamma_t^* + (1 - \gamma_t - \gamma_t^*) R_t \\ & - \delta_t(\pi) \frac{p_{t+1}}{p_t} - (1 + i_t^*) \delta_t^*(\pi) \end{aligned} \quad (19)$$

where  $i_t^*$  is the interest rate for borrowing in dollars. The maximization problem will be:

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<sup>9</sup>I will continue to assume that banks can borrow local currency at a zero interest rate.

<sup>10</sup>Morales [16] also talks about banks believing that Central Bank liquidity is of difficult availability, which was analyzed before.

$$\max_{\int_0^1} \left\{ \begin{array}{c} \pi \theta \ln(\alpha_t(\pi) \gamma_t + \delta_t(\pi)) + \\ \pi(1-\theta) \ln(\alpha_t^*(\pi) \gamma_t^* + \delta_t^*(\pi)) + \\ (1-\alpha_t(\pi)) \gamma_t \frac{p_{t+1}}{p_t} + (1-\alpha_t^*(\pi)) \gamma_t^* + \\ (1-\gamma_t-\gamma_t^*) R_t - \\ \delta_t(\pi) \frac{p_{t+1}}{p_t} - (1+i_t^*) \delta_t^*(\pi) \end{array} \right\} f(\pi) d\pi$$

subject to

$$\begin{array}{lll} 0 \leq \alpha_t(\pi) \leq 1 & 0 \leq \gamma_t \leq 1 & 0 \leq \delta_t(\pi) \\ 0 \leq \alpha_t^*(\pi) \leq 1 & 0 \leq \gamma_t^* \leq 1 & 0 \leq \delta_t^*(\pi) \end{array}$$

**Proposition 3** *When banks can borrow dollars at a positive interest rate, the equilibria of the model are unchanged. In particular for values of  $\pi \in [0, \pi^*]$  the solutions for  $\alpha$  and  $\alpha^*$  are given by equations (6) and (7) and for values of  $\pi \in [\pi^*, 1]$  the solutions for  $\delta$  and  $\alpha^*$  are given by equations (10) and (11).*

**Proof.** Following the same reasoning as in part 3.4, the first order conditions for  $\alpha$  and  $\alpha^*$  are:

$$\alpha_t(\pi) = \pi \theta \left[ 1 + \frac{\gamma_t^*}{\gamma_t} \frac{p_t}{p_{t+1}} + \frac{R_t}{\gamma_t} \frac{p_t}{p_{t+1}} (1 - \gamma_t - \gamma_t^*) - \frac{\delta_t^*(\pi)}{\gamma_t} \frac{p_t}{p_{t+1}} i_t^* \right] - \frac{\delta_t(\pi)}{\gamma_t} \quad (20)$$

$$\alpha_t^*(\pi) = \pi(1-\theta) \left[ 1 + \frac{\gamma_t}{\gamma_t^*} \frac{p_{t+1}}{p_t} + \frac{R_t}{\gamma_t^*} (1 - \gamma_t - \gamma_t^*) - \frac{\delta_t^*(\pi)}{\gamma_t^*} i_t^* \right] - \frac{\delta_t^*(\pi)}{\gamma_t^*} \quad (21)$$

As long as borrowing in dollars is even a little bit more expensive than borrowing in pesos, the bank will still want to hold enough dollars to cover the worst-case scenario, so that it will never have to borrow dollars. That is, the bank will always choose  $\delta^* = 0$ , because this type of borrowing is costly. Then substituting  $\delta^* = 0$  in equations (20) and (21), we get exactly the same equations (6) and (7). It is clearly shown that for the second part  $\alpha = 1$ ,  $\delta > 0$  and  $\delta^* = 0$ , so we get equations (10) and (11). (Q.E.D.)

Note that again for all values of  $\pi$ , the bank would be able to guarantee the same return to people (see equation 12) and propositions 1 and 2 still hold. In this way, it is seen that the results hold not only in the case where borrowing in dollars is impossible, but also in more general cases.

## 5 Conclusions

I have studied a pure-exchange economy in which spatial separation, limited communication and random relocation combine to create an environment for analyzing the dollarization of the banking system. It has been shown that banks will hold relatively large positions in dollars compared to the transactions demand for dollars. The results are the same whether the banks are unable to borrow dollars from the Central Bank or this type of borrowing is possible but costly. In either case, banks will choose to hold enough dollars to cover the worst case scenario; in other words, the first line of defense, in case of a liquidity crisis, will be provided by the banks themselves.

It has been shown also that dollarization of the banking system increases faster than the demand for dollars, and this is an equilibrium as long as  $\theta \in [\bar{\theta}, 1]$ . If  $\theta$  is much larger than  $\bar{\theta}$ , banks could be holding a lot of pesos. The model shows, however, that the dollarization of the banking system will be surprisingly high relative to the dollarization of the whole economy. That, is  $1 - \theta$  could be interpreted as a measure of what transactions are carried in dollars. In theory, it could be small or large, but whatever  $1 - \theta$  is, the banking system will be more dollarized than that because of the dollarization multiplier.

The model has been presented assuming that banks cannot borrow any dollars from the Central Bank, or can borrow but at a penalty rate. The latter case allows for more general results. In a crisis situation, when there is a shortage of dollars, it seems natural for a "dollar premium" to emerge (that is, for dollars to be at least slightly more expensive for a bank to get), especially if the Central Bank is using up all (or almost all) of its reserves in order to act as a liquidity insurer for the banking system.

Finally, I must mention that the model can be compared using country data. If someone looks at the data generated by the model, in each period, she would observe the realized demand for pesos and the realized demand for dollars. Just looking at that data, the banking system will probably appear to be "overly" dollarized. That is, it is likely that there will be realizations of the variable  $\pi$  that are above  $\pi^*$ , so that sometimes the bank will run out of pesos and need to borrow. But there will probably not be any realizations of  $\pi$  that are very close to 1. So even though the bank is occasionally running out of pesos, the bank will always have a stock of dollars around that it is not using. Why is the bank so "attached" to holding dollars even when there are regular shortages of pesos? The model tells us the answer: the demand for dollars is driven by "rare events" or the worst-case scenario. If we observe data from a period in which the worst case did not happen, the data will be "biased" in a sense.

Dollarization will be costly to the banks, insofar that they have to maintain more liquidity in dollars. It would be interesting to see how this could affect financial intermediation, the quantity of loans given, and the level of economic activity. These issues are left for future research.

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