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LAND PROTECTION, CREDIT ACCESS, AND THE INSURANCE GAP:

The De Soto Trap

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Abstract

Bolivia's Ley 1720 (April 2026) allows smallholders to voluntarily convert constitutionally protected *pequeña propiedad*—land that cannot be seized by creditors—into pledgeable *mediana propiedad*, explicitly targeting the credit-access channel that Hernando de Soto popularized. This paper develops a dynamic model showing that such conversion creates a *De Soto trap*: the households most likely to convert are those most desperate for liquidity, not those best positioned to benefit from credit, and these households face the highest default probability. Without complementary insurance, land flows irreversibly from vulnerable smallholders to those with capital to purchase foreclosed properties. The model yields three results: (i) adverse selection into conversion emerges endogenously from the threshold conditions; (ii) aggregate smallholder land decays exponentially toward zero—a ratchet that cannot reverse without policy intervention; (iii) mandatory insurance as a conversion requirement filters out non-viable applicants, bounds land loss, and raises net welfare by a factor of four. Calibrated to Bolivian agricultural census data, the model predicts that under current policy, 120,000 households will lose their land within 25 years. The law addresses the channel (credit) for which empirical evidence is weakest while removing a protection

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(implicit insurance through inalienability) whose value the risk literature suggests is large. Bolivia's reform inverts the optimal policy sequence.

JEL Codes: O12, O16, Q15, G21, G22, D86.
Keywords: land tenure, credit constraints, insurance, adverse selection, land reform, Bolivia.

Resumen

La Ley 1720 de Bolivia (abril de 2026) permite a los pequeños agricultores convertir voluntariamente la «pequeña propiedad» -protegida constitucionalmente y que no puede ser embargada por los acreedores- en «mediana propiedad» pignorable, centrándose explícitamente en el canal de acceso al crédito que popularizó Hernando de Soto. Este documento desarrolla un modelo dinámico que muestra que dicha conversión crea una trampa de De Soto: los hogares con mayor probabilidad de convertir son aquellos más desesperados por obtener liquidez, no los mejor posicionados para beneficiarse del crédito, y estos hogares enfrentan la mayor probabilidad de incumplimiento. Sin un seguro complementario, la tierra fluye irreversiblemente de los pequeños agricultores vulnerables hacia aquellos con capital para comprar propiedades embargadas. El modelo arroja tres resultados: (i) la selección adversa hacia la conversión surge de manera endógena de las condiciones de umbral; (ii) la tierra agregada de los pequeños agricultores se reduce exponencialmente hacia cero -un efecto de trinquete que no puede revertirse sin intervención política-; (iii) el seguro obligatorio como requisito de conversión filtra a los solicitantes no viables, limita la pérdida de tierras y aumenta el bienestar neto en un factor de cuatro. Calibrado con datos del censo agrícola boliviano, el modelo predice que, bajo la política actual, 120 000 hogares perderán sus tierras en un plazo de 25 años. La ley aborda el canal (el crédito) para el que la evidencia empírica es más débil, al tiempo que elimina una protección (el seguro implícito a través de la inalienabilidad) cuyo valor, según sugiere la literatura sobre riesgos, es considerable. La reforma de Bolivia invierte la secuencia óptima de políticas.

Código JEL: O12, O16, Q15, G21, G22, D86.
Palabras clave: propiedad de la tierra, restricciones crediticias, seguros, selección adversa, reforma agraria, Bolivia.

1 Introduction

> "Nuestras tierras no están sujetas a préstamos, no son mercancía, sino vida."
>
> —Javier Fernández, President of CIPOAP, April 2026

On April 10, 2026, Bolivia enacted Ley 1720, authorizing the National Agrarian Reform Institute (INRA) to convert *pequeña propiedad* titles into *mediana propiedad* titles upon voluntary request. The conversion requires only a written application and a sworn declaration; it is free, processed within ten business days, and irreversible. Within five days of promulgation, indigenous and campesino organizations from Pando, Beni, and the highlands declared a national emergency, launched marches toward La Paz, and gave the government 48 hours to abrogate the law. Eleven civil society organizations—including Fundación Tierra, CIPCA, and CEJIS—announced an immediate constitutional challenge, calling the law "unconstitutional and regressive" and warning it would trigger "a new stage of land reconcentration."

The conflict crystallizes a tension at the heart of property rights theory. Bolivia's 2009 Constitution declares *pequeña propiedad* to be "indivisible, constitutes inalienable family patrimony, and is not subject to seizure" (Article 394-II). This protection eliminates the possibility of foreclosure—but also eliminates the possibility of using land as collateral. Ley 1720 reverses this: conversion to *mediana propiedad* makes land fully alienable and pledgeable, enabling access to mortgage credit but exposing the owner to foreclosure risk. The government frames the reform as "unlocking capital" for rural development; opponents frame it as opening the door to dispossession.

Both sides invoke Hernando de Soto. Proponents cite *The Mystery of Capital*: untitled or informally held property constitutes "dead capital" that formal titling could unlock for collateral-based lending. Opponents cite the same book's premise—that secure property rights are foundational to development—to argue that removing constitutional protections undermines security rather than enhancing it. The empirical literature offers no easy resolution. Three decades of research have produced a consistent finding: property rights reforms reliably increase investment through the tenure-security channel but rarely improve credit access for the poor.

This paper provides a theoretical framework for analyzing such voluntary conversion policies. The core insight is that protected tenure functions as implicit insurance. A household under *pequeña propiedad* faces income risk from weather, prices, and health shocks, but it cannot lose its land regardless of how severe or persistent those shocks are. The protection is perfect (covering all states of the world), costless (no premium), and automatic (requiring no action). Removing protection is equivalent to canceling this insurance policy. The question is what replaces it. In the absence of explicit insurance markets, nothing does. The household gains access to credit but bears uninsured land-loss risk. This creates a selection problem: which households choose to bear this risk?

We prove that without insurance, conversion exhibits adverse selection. The households most likely to convert are not those best positioned to benefit from credit (high productivity, low risk) but those most desperate for liquidity (low wealth, high discount rates) or most uninformed about risk (those who underestimate default probability). These households are precisely those most likely to default and lose their land. The result is what we call the *De Soto trap*: a policy designed to unlock capital instead triggers a ratchet that transfers land from vulnerable smallholders to those with capital to purchase foreclosed properties.

1.1 The Bolivian Context

Bolivia's agrarian structure makes it an especially consequential laboratory for this question. The 2013 Agricultural Census recorded 871,927 agricultural production units (UPAs) on 34.7 million hectares. The distribution is extremely skewed: mean gross production value per UPA is Bs 248,000 but the median is Bs 8,246—a 30-fold ratio. Half of all UPAs sit in departments dominated by communal and TCO (Tierra Comunitaria de Origen) tenure—La Paz, Oruro, Potosí, Pando—while 16 percent occupy the commercial lowland bloc of Santa Cruz and Beni. This 16 percent farms 67 percent of total agricultural land.

The capital intensity gap between tenure regimes is stark. Santa Cruz has 218 tractors per 1,000 UPAs; La Paz has 9. Mean gross production value in the "unprotected-dominant" bloc (Santa Cruz, Beni) is Bs 512,000 per UPA; in the "protected-dominant" bloc (La Paz, Oruro, Potosí, Pando) it is Bs 9,200—a 56-fold gap. Even after controlling for land area, livestock, and irrigation, a residual 0.77 log-unit productivity premium remains for unprotected tenure. Proponents of Ley 1720 cite this gap as evidence that collateral constraints bind and that conversion will close it.

The opponents' counter-argument is equally grounded in data. The 2015 Agricultural Survey shows that credit access rates are nearly identical across tenure regimes: 13.2 percent of indigenous-communal UPAs obtained credit, versus 14.7 percent of individual UPAs and 14.3 percent of empresarial/cooperative UPAs. The gap appears not in credit access but in what credit buys: tractor *ownership* rates are 2.9 percent for indigenous-communal UPAs versus 52 percent for empresarial UPAs—an 18-fold ratio. Households under protected tenure can rent tractors (28.8 percent do), but ownership requires either financing or inheritance. The collateral constraint appears to bind at the asset-ownership margin, not the credit-extension margin.

This pattern is consistent with two mechanisms that generate very different policy implications. Under the first, protected tenure causes the capital gap: inalienability prevents collateral-backed investment loans, so conversion would raise productivity. Under the second, protected tenure is correlated with but does not cause the gap: agroecology, market access, and historical asset accumulation explain the differential, so conversion would expose households to foreclosure risk without raising productivity. Only data on credit rejection reasons can distinguish these mechanisms—and Bolivia's agricultural surveys do not collect it.

1.2 The Political Economy

The timing of Ley 1720 is not incidental. Bolivia entered 2026 in macroeconomic crisis: foreign reserves depleted, the parallel exchange rate at twice the official rate, fuel shortages, and GDP projected to contract 3.3 percent. The government of Rodrigo Paz, inaugurated in November 2025 after a fragmented first-round election and a contested runoff, faces fiscal constraints that preclude large-scale public investment. Credit access through private channels offers a politically attractive alternative: it promises rural development without budgetary cost.

The law emerged from Santa Cruz, Bolivia's agroindustrial heartland. President Paz signed it at Agropecruz, the annual showcase of elite cattle and commercial grain. Fundación Tierra's director, Juan Pablo Chumacero, observed that the primary beneficiaries would be "agroindustry in the east and medium-property holders"—not the smallholders the law nominally targets. The 11-organization coalition opposing the law warned that it was "not an isolated measure but the first step in a package of agrarian laws aimed at fragmenting communal lands and indigenous territories."

Indigenous opposition centers on two concerns. First, the constitutional architecture: Article 394-II's declaration that *pequeña propiedad* is "inembargable" was not incidental but constitutive—a codification of the 1953 Agrarian Reform's central achievement. Conversion to *mediana propiedad* does not merely reclassify land; it removes it from the constitutional protection that defines the agrarian regime. Second, the voluntariness premise: opponents argue that "voluntary" conversion in a context of economic crisis, high interest rates, and asymmetric information between banks and smallholders will produce adverse selection—the same mechanism this paper formalizes.

1.3 Contribution

This paper makes three contributions.

First, we provide a dynamic model that formalizes the insurance-credit tradeoff implicit in conversion policies. The model features endogenous selection into conversion, credit-financed investment with default risk, and irreversible land loss creating a ratchet dynamic. We prove that adverse selection arises endogenously from the threshold conditions, not as an assumption: low-wealth households face lower productivity thresholds for conversion (credit is more valuable when constraints bind tightly) and convert despite high default probability. The result is that converters have higher average default probability than the population, accelerating land concentration.

Second, we extend the model to include labor reallocation and rental markets—the channels that **?** identify as primary in Mexico's ejido reform. The rental option provides gains from conversion that are separate from credit access and substitutes partially for the insurance value of inalienability. With rental, adverse selection is mitigated, the ratchet slows, and welfare gains from conversion increase. Bolivia's law, however, does not create a rental market; it creates only a credit market. The extension clarifies what is missing.

Third, we calibrate the model to Bolivian microdata and generate quantitative predictions. Bolivia's land Gini is already 0.914—among the highest in Latin America—with 0.4% of holdings controlling 47% of agricultural land. Under the law as passed (no insurance requirement), we predict that within 25 years, **192,000 households** will lose their land through foreclosure. Under a counterfactual with mandatory insurance, land loss falls by 78 percent (to 42,000 households). Net welfare among converters is *negative* (-33%) without insurance, but positive ($+17\%$) with insurance. The law as passed is welfare-destroying; a mandatory insurance requirement would transform it into welfare-improving. These predictions are testable: within 5–10 years, conversion rates, credit uptake, and early defaults will reveal which scenario is unfolding.

1.4 Related Literature

This paper connects three literatures that have developed largely in isolation.

The property rights and credit literature, following **?**, emphasizes that untitled or inalienable property cannot serve as collateral, creating "dead capital" that suppresses investment. **?** formalized three channels: tenure security, collateral/credit, and gains from trade. The empirical record has increasingly qualified the credit channel. **?** found that Buenos Aires titling increased housing investment but had "modest" credit effects and no labor income effects; they concluded that titling reduces poverty "not through the shortcut of credit access, but through the slow channel of increased physical and human capital investment." **?** found that Peru's COFOPRI program increased labor supply through reduced "guard labor" rather than credit effects. **?** found no increase in private-sector bank lending to titled households. **?** showed that in Paraguay, titling amplified credit supply only for wealthier producers; for smaller farms, the increase in collateral wealth was insufficient to relax credit constraints. The **?** meta-analysis of 20 studies concluded that "the credit channel finds no support."

The agricultural risk and insurance literature documents pervasive uninsured risk in developing-country agriculture (**??**) and the welfare costs of missing insurance markets (**??**). **?** showed that insurance matters more than cash for investment decisions in Ghana. **?** identified "risk rationing" as distinct from quantity rationing: farmers who could borrow choose not to because collateral contracts offer lower expected well-being than safe subsistence. This predicts that wealthier, more diversified farmers will borrow after conversion while the poorest will self-exclude.

The land markets and concentration literature studies how land sales and foreclosures redistribute ownership over time (**???**). **?** analyzed Mexico's 1992 ejido reform—the closest parallel to Ley 1720—and found that the primary gains came through labor reallocation and migration, not credit. Critically, Mexico's reform included community-level safeguards: ejido assemblies retained veto power over land sales, creating a collective check on individual decisions. Bolivia's law has no such safeguard.

Our contribution is to model the dynamic consequences of making land pledgeable in a context of missing insurance markets, deriving the selection and ratchet effects formally and showing how policy design (insurance, buyer restrictions, buyback funds) affects outcomes. We show that the De Soto hypothesis can be internally consistent—pledgeability can raise productivity for those who convert and succeed—while still being welfare-reducing in expectation because the selection is adverse and the land loss is irreversible.

1.5 Roadmap

Section 2 presents the model primitives: households, technology, shocks, and the two tenure regimes. Section 3 analyzes the conversion decision and proves the selection results. Section D.7 derives the ratchet theorem with endogenous entry. Section D.8 introduces insurance and shows how it changes selection and dynamics. Section 6 extends the model to include labor reallocation and rental markets. Section 7 calibrates to Bolivia and generates quantitative predictions. Section B derives optimal policy and compares to Ley 1720. Section C concludes.

2 Model

Consider an economy with a continuum of agricultural households indexed by $i \in [0, 1]$. Each household owns one unit of land and chooses consumption, investment, and borrowing over an infinite horizon. Households are heterogeneous in productivity A_i, initial wealth W_i, and exposure to income risk σ_i. We denote a household's type as $\theta_i = (A_i, W_i, \sigma_i)$, drawn from a joint distribution $F(\theta)$ at $t = 0$.

2.1 Tenure Regimes

Land operates under one of two tenure regimes. Under *protected tenure*, land is inalienable and cannot be seized to satisfy debts. This regime corresponds to Bolivia's *pequeña propiedad*, which Article 394-II of the Constitution declares "indivisible, constitutes inalienable family patrimony, and is not subject to seizure." Under *unprotected tenure*, land is fully alienable and can be pledged as collateral. This regime corresponds to *mediana propiedad*, to which Ley 1720 permits voluntary conversion.

The critical distinction: protected households cannot borrow against land; unprotected households can, but they face foreclosure risk. At $t = 0$, each household chooses whether to convert from protected to unprotected tenure. Conversion is voluntary, costless, and irreversible.

2.2 Technology and Shocks

Households produce output using a Cobb-Douglas technology:

$$y_{it} = A_i k_{it}^{\alpha} L^{1-\alpha} \varepsilon_{it}, \tag{1}$$

where k_{it} is capital (equipment, inputs), $L = 1$ is land, $\alpha \in (0, 1)$ is the capital share, and ε_{it} is a multiplicative shock. The productivity parameter A_i is time-invariant and captures soil quality, farmer ability, and market access.

The shock ε_{it} has two components:

$$\ln \varepsilon_{it} = \eta_t + \nu_{it}, \quad \eta_t \sim N(0, \sigma_\eta^2), \quad \nu_{it} \sim N(0, \sigma_i^2). \tag{2}$$

The aggregate component η_t (weather, prices) affects all households; the idiosyncratic component ν_{it} (localized pests, illness) is household-specific. Exposure to idiosyncratic risk varies across households: σ_i may be high for rainfed plots in erosion-prone areas and low for irrigated plots with diversified crops.

2.3 Preferences and Timing

Households have CRRA preferences over consumption:

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{it}) \right], \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{3}$$

where $\beta \in (0, 1)$ is the discount factor and $\gamma > 0$ is relative risk aversion. A subsistence floor $\underline{c} > 0$ imposes a minimum consumption requirement.

Each period proceeds in five stages: (i) the aggregate shock η_t is realized; (ii) households choose capital k_{it} and borrowing b_{it}; (iii) idiosyncratic shocks ν_{it} are realized; (iv) production occurs and debts are due; (v) households that cannot service debt default.

2.4 Borrowing Constraints

Households can borrow from competitive lenders at gross interest rate $R > 1$. The borrowing limit depends on pledgeable collateral:

$$\text{Protected:} \quad b_{it} \leq 0, \tag{4}$$

$$\text{Unprotected:} \quad b_{it} \leq \phi P^L, \tag{5}$$

where P^L is the market price of land and $\phi \in (0, 1)$ is the loan-to-value ratio.

Protected households cannot borrow: land is their only significant asset, and it cannot be pledged. This constraint is the cost of protection. Unprotected households can borrow up to a fraction ϕ of their land value. This access is the benefit of conversion.

2.5 Default

A household with debt $b > 0$ defaults if realized income plus wealth cannot cover debt service and subsistence:

$$y_{it} + W_{it} < Rb_{it} + \underline{c}. \tag{6}$$

Upon default, the lender seizes the land and sells it at market price P_t^L. The household becomes landless, receiving continuation value V^L—the expected utility from wage labor or urban migration.

Assumption 1 (Landlessness is absorbing)**.** *A household that loses its land cannot reacquire it: V^L is a constant continuation value with no transition back to landownership.*

This assumption reflects the same credit constraint that motivates conversion: a landless household has no collateral with which to finance land purchase and insufficient savings to buy outright. The assumption is strong but empirically grounded: land markets in Bolivia are thin, prices are high relative to agricultural incomes, and landless households rarely reenter ownership.

2.6 Land Market

Foreclosed land is sold at auction. The price clears the market given supply (defaults D_t) and demand (from surviving households with excess wealth, outside investors, or agribusiness):

$$P_t^L = \bar{P}^L \cdot \left(\frac{\bar{D}}{D_t}\right)^{\theta}, \tag{7}$$

where $\bar{P}^L$ is the baseline price, $\bar{D}$ is a reference default quantity, and $\theta > 0$ governs price elasticity. When many households default simultaneously (high D_t), prices crash. This fire-sale mechanism amplifies the ratchet during aggregate downturns.

2.7 Value Functions

We characterize household behavior through value functions. Under **protected tenure**, the household solves:

$$V^P(\theta) = \max_{c,k} \left\{ u(c) + \beta \mathbb{E} \left[V^P(A, W') \right] \right\} \tag{8}$$

subject to $c + k = W$, $W' = Ak^\alpha \varepsilon$, and $c \geq \underline{c}$. The household allocates wealth between consumption and self-financed investment. Because $b = 0$, no default can occur: the household retains land in all states.

Under **unprotected tenure**, the household solves:

$$V^U(\theta) = \max_{c,k,b} \left\{ u(c) + \beta \mathbb{E} \left[(1 - \mathbf{1}_D) V^U(A, W') + \mathbf{1}_D V^L \right] \right\} \tag{9}$$

subject to $c + k = W + b$, $b \leq \phi P^L$, $W' = Ak^\alpha \varepsilon - Rb$, and $c \geq \underline{c}$. The indicator $\mathbf{1}_D$ equals one if equation (6) holds. The household now has access to credit but faces the risk of losing land.

Proposition 1 (Existence and Uniqueness)**.** *The value functions V^P and V^U exist, are unique, bounded, continuous, and strictly increasing in A and W.*

Proof. Both Bellman operators are contractions on the space of bounded continuous functions under the sup norm. Boundedness follows from $u(c) \leq u(\bar{c})$ for some upper bound $\bar{c}$ implied by the resource constraint. Monotonicity in A and W follows from the fact that higher A raises output for any capital choice and higher W relaxes the budget constraint. □

2.8 The Insurance Interpretation

Protected tenure provides implicit insurance against land loss. The insurance is complete (covering all states, including catastrophic shocks), costless (no premium), and automatic (requiring no action). We formalize this as:

$$V^P = V^{\text{expected}} + \Omega^{\text{insurance}}, \tag{10}$$

where V^{expected} is the value of the expected income stream and $\Omega^{\text{insurance}} > 0$ is the additional value from eliminating land-loss risk for any risk-averse household ($\gamma > 0$).

Conversion removes this insurance. The household gains access to credit but bears uninsured land-loss risk. Without explicit insurance markets, nothing replaces the implicit protection. This tradeoff—credit access versus land security—drives the selection patterns we analyze in Section 3.

2.9 Underinvestment under Protection

The first-order condition for capital under protected tenure is:

$$u'(c) = \beta \mathbb{E}\left[V_W^P(A, W') \cdot \alpha A k^{\alpha-1} \varepsilon\right]. \tag{11}$$

Because the household cannot borrow, investment is constrained by $k \leq W - \underline{c}$. If the unconstrained optimum k^* exceeds this bound, the household is credit-constrained: its marginal product of capital exceeds the cost of capital.

Corollary 1 (Credit Constraint). *Households with $W - \underline{c} < k^*(A)$ underinvest relative to first-best. The underinvestment is more severe for lower-wealth households (tighter constraint) and higher-productivity households (higher optimal k^*).*

This underinvestment is the cost of protection that motivates conversion. High-productivity, low-wealth households gain most from credit access: their marginal product is high, their constraint binds tightly, and relaxing it raises output substantially. But these same households may face high default risk if their income variance is large. Whether they gain or lose from conversion depends on the balance—a question we formalize in the next section.

3 The Conversion Decision

At $t = 0$, each household compares the value of protected tenure $V^P(\theta)$ to the value of unprotected tenure $V^U(\theta)$ and converts if and only if $V^U > V^P$. The conversion gain is:

$$\Delta(\theta) \equiv V^U(A, W, \sigma) - V^P(A, W). \tag{12}$$

A household converts if $\Delta > 0$. We characterize who converts and what this implies for default incidence.

3.1 Decomposing the Conversion Gain

The conversion gain has two components: the benefit of credit access and the cost of default risk.

Proposition 2 (Decomposition)**.** *The conversion gain admits the representation:*

$$\Delta(A, W, \sigma) = \underbrace{B(A, W)}_{\text{credit benefit}} - \underbrace{L(A, W, \sigma)}_{\text{expected land loss}}, \tag{13}$$

where $B(A, W)$ is the expected utility gain from credit-financed investment conditional on no default, and $L(A, W, \sigma)$ is the expected loss from land forfeiture weighted by default probability.

Proof. Write V^U as the sum over default and non-default paths:

$$V^U = \sum_{t=0}^{\infty} \beta^t \left[(1 - p_t) \cdot \mathbb{E}[u(c_t^U)|\text{survive}] + p_t \cdot V^L \right]. \tag{14}$$

Subtracting $V^P = \sum_{t=0}^{\infty} \beta^t \mathbb{E}[u(c_t^P)]$ yields the decomposition. □

The components have opposite signs in the relevant arguments. B is increasing in A (high productivity raises returns to investment) and ambiguous in W (credit is more valuable when the constraint binds, but relaxing a non-binding constraint yields little). L is increasing in σ (higher risk raises default probability), decreasing in A (higher productivity reduces default threshold), and decreasing in W (wealth provides a buffer).

3.2 The Threshold Surface

Definition 1 (Conversion Threshold)**.** *For each* (W, σ)*, define:*

$$\bar{A}(W, \sigma) = \inf\{A : \Delta(A, W, \sigma) \geq 0\}. \tag{15}$$

Households with $A \geq \bar{A}(W, \sigma)$ *convert; those with* $A < \bar{A}(W, \sigma)$ *remain protected.*

The threshold surface $\bar{A}(W, \sigma)$ partitions the type space into converters and non-converters.

Proposition 3 (Threshold Properties)**.** *The threshold surface satisfies:*

(i) $\partial \bar{A} / \partial \sigma > 0$*: higher risk requires higher productivity to justify conversion.*

(ii) For W *below some* W^*, $\partial \bar{A} / \partial W < 0$*: severely constrained households face lower productivity thresholds.*

(iii) For all (W, σ)*, sufficiently high* A *guarantees conversion.*

Proof. (i): By the implicit function theorem on $\Delta(\bar{A}, W, \sigma) = 0$:

$$\frac{\partial \bar{A}}{\partial \sigma} = -\frac{\partial \Delta / \partial \sigma}{\partial \Delta / \partial A} = -\frac{-\partial L / \partial \sigma}{\partial B / \partial A - \partial L / \partial A} > 0, \tag{16}$$

since $\partial L / \partial \sigma > 0$ and the denominator is positive.

(ii): When credit constraints bind severely, the marginal value of credit is high: $\partial B / \partial W < 0$ (loosening the constraint when it binds tightly is valuable). This makes $\partial \Delta / \partial W > 0$, so higher W raises Δ and the threshold $\bar{A}$ that sets $\Delta = 0$ must fall. Equivalently, low-W households need lower productivity to find conversion worthwhile.

(iii): As $A \to \infty$, $B \to \infty$ and $L \to 0$ (high productivity dominates default risk). Thus $\Delta \to \infty > 0$. □

Property (ii) is the key to adverse selection. Low-wealth households face a *lower* bar for conversion—not because they are well-suited to credit, but because their outside option (remaining protected and credit-constrained) is poor.

3.3 Two Types of Selection

Selection operates on multiple margins. Define:

$$\text{Selection}_A = \mathbb{E}[A|\mathcal{C}] - \mathbb{E}[A], \tag{17}$$

$$\text{Selection}_p = \mathbb{E}[p|\mathcal{C}] - \mathbb{E}[p], \tag{18}$$

where $\mathcal{C} = \{(A, W, \sigma) : \Delta > 0\}$ is the converting population and $p(A, W, \sigma)$ is the default probability under optimal unprotected-tenure policy.

Positive Selection_A means converters have higher productivity than the population. Positive Selection_p means converters have higher default probability—adverse selection in the credit-market sense.

3.4 Positive Selection on Productivity

Theorem 1 (Selection on A)**.** *Conditional on (W, σ), converters have higher productivity than non-converters:*

$$\mathbb{E}[A|\mathcal{C}, W, \sigma] > \mathbb{E}[A|W, \sigma]. \tag{19}$$

Proof. For fixed (W, σ), conversion requires $A \geq \bar{A}(W, \sigma)$. Therefore $\mathbb{E}[A|\mathcal{C}, W, \sigma] = \mathbb{E}[A|A \geq \bar{A}]$, which exceeds $\mathbb{E}[A]$ by truncation from below. □

This is positive selection—but it is conditional on wealth and risk exposure. The unconditional selection depends on who shows up at the threshold. If low-W households are over-represented among converters (because their threshold is low), and low-W correlates with high p, then adverse selection on p can coexist with positive selection on A.

3.5 Adverse Selection on Default Probability

Theorem 2 (The Desperation Mechanism)**.** *Suppose the joint distribution $F(A, W, \sigma)$ satisfies:*

(C1) ***Weak or negative wealth-productivity correlation:*** $\text{Cov}(A, W) \leq 0$*.*

(C2) ***Credit constraint salience:*** *For low-W households, the credit benefit $B(A, W)$ is large relative to the expected land loss $L(A, W, \sigma)$.*

(C3) ***Desperation converts:*** *Low-W households have $\bar{A}(W, \sigma)$ low enough that they convert despite moderate A.*

Then the average default probability among converters exceeds the population average:

$$\mathbb{E}[p|\mathcal{C}] > \mathbb{E}[p]. \tag{20}$$

Proof. Partition the type space by wealth. Let W^* be the threshold below which credit constraints bind severely.

Step 1: Default probability comparative statics. From $p = G(\varepsilon^*; \sigma)$ where $\varepsilon^* = (Rb + \underline{c})/(Ak^\alpha)$: (i) $\partial p/\partial A < 0$ (higher productivity reduces the default threshold); (ii) $\partial p/\partial W < 0$ (higher wealth reduces borrowing needs); (iii) $\partial p/\partial \sigma > 0$ (higher risk fattens the left tail).

Step 2: Threshold is lower for low-W households. By Proposition 3(ii), when $W < W^*$, we have $\partial \bar{A}/\partial W < 0$. Low-$W$ households face a lower productivity bar for conversion.

Step 3: Composition of converting population. Partition converters:

$$\mathcal{C} = \mathcal{C}_{\text{low}} \cup \mathcal{C}_{\text{high}}, \quad \mathcal{C}_{\text{low}} = \mathcal{C} \cap \{W < W^*\}. \tag{21}$$

High-W converters have low p: they convert because A is high (Theorem 1), and high A combined with high W reduces default probability.

Low-W converters have high p: they convert despite moderate A because the credit benefit is large when constraints bind (C2). But moderate A combined with low W means high default probability. Under (C1), low-W households are not systematically high-A, so the productivity offset is limited.

Step 4: Aggregation. The average p in $\mathcal{C}$ is:

$$\mathbb{E}[p|\mathcal{C}] = \frac{\Pr(\mathcal{C}_{\text{low}})}{\Pr(\mathcal{C})}\mathbb{E}[p|\mathcal{C}_{\text{low}}] + \frac{\Pr(\mathcal{C}_{\text{high}})}{\Pr(\mathcal{C})}\mathbb{E}[p|\mathcal{C}_{\text{high}}]. \tag{22}$$

Under (C1)–(C3), $\Pr(\mathcal{C}_{\text{low}})$ is non-negligible and $\mathbb{E}[p|\mathcal{C}_{\text{low}}]$ is high. The non-converting population includes high-W, low-A households (who would have low p but fail the threshold) and high-σ households (screened out by the rising threshold). These tend to have lower p than the low-W desperate converters. The composition effect yields $\mathbb{E}[p|\mathcal{C}] > \mathbb{E}[p]$. □

The theorem provides a mechanism for adverse selection that is *derived*, not assumed. The key is the desperation channel: low-W households convert not because credit is productive for them but because their alternative—remaining protected with a binding credit constraint—is bad. These households drag up the average default probability among converters.

3.6 When Adverse Selection Fails

Proposition 4 (Positive Selection Regime)**.** *If* $\text{Cov}(A, W) > 0$ *is strongly positive and credit constraints bind weakly (high average* W*), then:*

$$\mathbb{E}[p|\mathcal{C}] < \mathbb{E}[p]. \tag{23}$$

Conversion exhibits positive selection: only high-A*, high-*W*, low-*σ *households convert, and these have low default probability.*

Proof. When W is high on average, credit constraints bind weakly. The conversion benefit $B(A, W)$ is small for most households. Only very high-A households—for whom the productivity gain from credit-financed investment is large—find conversion worthwhile. These households have low p. When $\text{Cov}(A, W) > 0$, high-A households also tend to have high W, compounding the low-p effect. □

The two regimes—adverse versus positive selection—depend on the wealth distribution and the correlation structure of household types.

3.7 Bolivia's Position

Three features of the Bolivian context favor the adverse selection regime.

First, Bolivia's 1953 agrarian reform and subsequent land distributions created smallholdings largely independent of productivity. The assignment mechanism—expropriation of hacienda land and distribution to former laborers and settlers—broke any pre-existing correlation between A and W. If anything, households on marginal land (low A) received smaller plots (low W), suggesting $\text{Cov}(A, W) \leq 0$.

Second, the 2024–2026 macroeconomic crisis has depleted household wealth. With foreign reserves exhausted, the parallel exchange rate at twice the official rate, fuel shortages, and GDP contracting 3.3%, agricultural households have drawn down savings and livestock to cover consumption. Credit constraints now bind for a larger share of the population, making the conversion benefit $B(A, W)$ salient for more households.

Third, Ley 1720 imposes no screening. The conversion process requires only a written application and a sworn declaration; it is processed in ten days with no cost, no insurance requirement, no financial literacy test, and no cooling-off period. This design selects in all households for whom $\Delta > 0$, including the desperate low-W households that a well-designed program would screen out.

Under these conditions, Theorem 2 applies: the converting population will have higher average default probability than the general population. The ratchet will operate with adverse selection, accelerating land concentration.

3.8 Selection Intensity

Definition 2 (Adverse Selection Intensity)**.** *Define:*

$$\mathcal{I} = \frac{\mathbb{E}[p|\mathcal{C}] - \mathbb{E}[p]}{\sqrt{\text{Var}(p)}}. \tag{24}$$

The intensity $\mathcal{I}$ measures how far the converter distribution shifts relative to the population spread.

Proposition 5 (Determinants of Selection Intensity)**.** *Under the conditions of Theorem 2:*

(i) $\mathcal{I}$ increases with dispersion in W: greater wealth inequality creates a larger pool of desperate converters.

(ii) *$\mathcal{I}$ increases as constraints become more binding: lower average W raises the credit benefit for constrained households.*

(iii) *$\mathcal{I}$ decreases with $\mathrm{Cov}(A, W)$: positive correlation between productivity and wealth offsets the desperation mechanism.*

Bolivia's high rural wealth inequality, binding credit constraints, and weak A-W correlation all predict high selection intensity. The model predicts that $\mathbb{E}[p|\mathcal{C}]$ will substantially exceed the population mean $\mathbb{E}[p]$—a prediction testable once conversion data become available.

3.9 Graphical Representation

Figure 1 illustrates the threshold surface and selection patterns.

The figure makes visible the core mechanism: the threshold surface is not horizontal in W. For credit-constrained households, the threshold dips, pulling in low-W types who would otherwise remain protected. These households raise the average default probability among converters.

4 Ratchet Dynamics

Conversion is a one-time decision; default unfolds over time. This section characterizes the evolution of land ownership, proves that converted land decays to zero in expectation, and shows how adverse selection and correlated shocks accelerate the process.

4.1 Land Accounting

Partition total land into three categories:

$$N_t^P = \text{protected land (non-converters)}, \tag{25}$$

$$N_t^U = \text{unprotected land (converters who have not yet defaulted)}, \tag{26}$$

$$N_t^C = \text{concentrated land (acquired through foreclosure purchase)}. \tag{27}$$

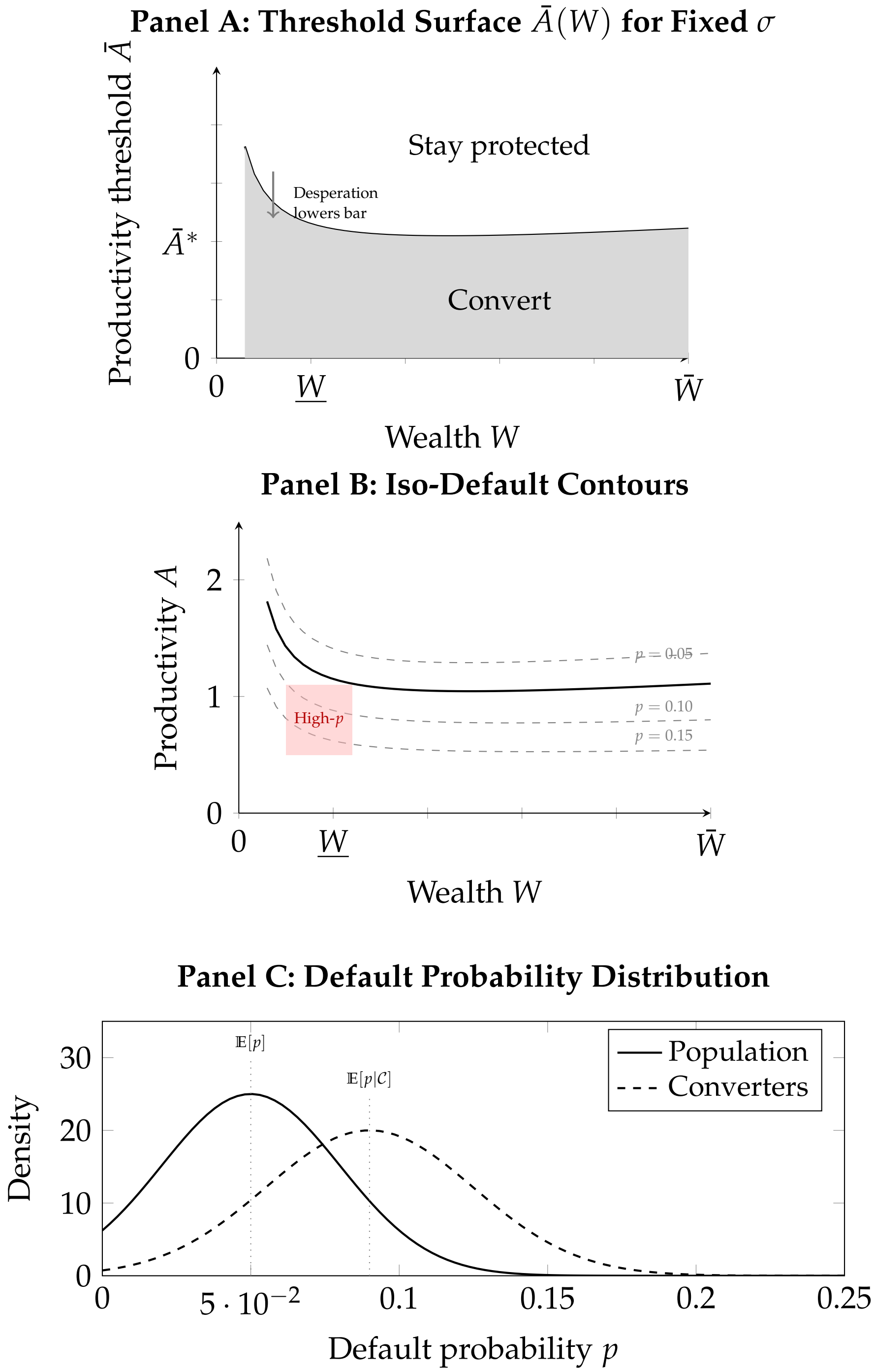

Figure 1: Threshold surface and selection into conversion. Panel A: threshold productivity $\bar{A}(W)$ for fixed risk σ. The threshold dips at low wealth (credit more valuable when constraints bind), pulling in "desperate" households. Panel B: iso-default-probability contours overlaid on conversion region. Low-W converters (shaded) cross high-p contours. Panel C: distribution of default probability in the full population versus among converters. Under adverse selection, the converter distribution is shifted right: $\mathbb{E}[p|\mathcal{C}] > \mathbb{E}[p]$.

At $t = 0$, after conversion decisions:

$$N_0^P = 1 - \lambda, \quad N_0^U = \lambda, \quad N_0^C = 0, \tag{28}$$

where $\lambda = \Pr(\mathcal{C})$ is the conversion rate.

Each period, some unprotected households default. Let D_t denote the measure of defaults at time t. The transition equations are:

$$N_{t+1}^P = N_t^P, \tag{29}$$

$$N_{t+1}^U = N_t^U - D_t, \tag{30}$$

$$N_{t+1}^C = N_t^C + D_t. \tag{31}$$

Protected land is permanent: equation (29) has no loss term. Unprotected land shrinks with each default: equation (30) is monotonically decreasing. Concentrated land grows: equation (31) accumulates all foreclosed properties.

4.2 The Ratchet Theorem

Let p_t denote the average default probability among surviving unprotected households at time t:

$$p_t = \mathbb{E}[p(A, W_t, \sigma) \mid \text{survived through } t]. \tag{32}$$

Theorem 3 (Exponential Decay). *Under the model of Section 2:*

(i) The expected measure of unprotected land decays exponentially:

$$\mathbb{E}[N_t^U] = N_0^U \cdot \prod_{s=0}^{t-1}(1 - p_s). \tag{33}$$

(ii) If $p_s \geq \underline{p} > 0$ for all s, then:

$$\mathbb{E}[N_t^U] \leq N_0^U \cdot (1 - \underline{p})^t \to 0 \text{ as } t \to \infty. \tag{34}$$

(iii) All converted land eventually transitions to concentrated holdings:

$$\lim_{t\to\infty} \mathbb{E}[N_t^C] = N_0^U = \lambda. \tag{35}$$

Proof. Define the survival probability through period t:

$$S_t = \prod_{s=0}^{t-1}(1 - p_s). \tag{36}$$

(i) A household that converted at $t = 0$ retains its land at t if and only if it has not defaulted in any period $s \in \{0, \ldots, t-1\}$. The probability of this event is S_t. Aggregating across converters: $\mathbb{E}[N_t^U] = N_0^U \cdot S_t$.

(ii) If $p_s \geq \underline{p}$ for all s, then $1 - p_s \leq 1 - \underline{p} < 1$, so:

$$S_t \leq (1 - \underline{p})^t \to 0 \text{ as } t \to \infty. \tag{37}$$

(iii) Since $N_t^P + N_t^U + N_t^C = 1$ and $N_t^P = N_0^P$ is constant, we have $N_t^C = 1 - N_0^P - N_t^U = N_0^U - N_t^U$. As $N_t^U \to 0$, $N_t^C \to N_0^U$. □

The theorem establishes that the ratchet cannot reverse without policy intervention. Every period, some unprotected households default. The land they lose is purchased by agents outside the smallholder sector. No mechanism returns this land to smallholder ownership (by Assumption 1). The process is monotonic and, given positive default probability, converges to complete displacement.

4.3 Duration Dependence

The default probability p_t need not be constant over time. Two forces operate in opposite directions.

Positive duration dependence (falling p_t): High-risk households default early, leaving a lower-risk surviving pool. The average σ among survivors falls; their average p falls.

Negative duration dependence (rising p_t): Wealth depletion. Each period, households draw down wealth to cover consumption and debt service. After several bad shocks, a household's buffer W_t shrinks. Lower W_t raises p.

Proposition 6 (Net Duration Dependence)**.** *(i) If wealth dynamics dominate selection effects, p_t is increasing in t (negative duration dependence). The ratchet accelerates.*

(ii) If selection effects dominate wealth dynamics, p_t is decreasing in t (positive duration dependence). The ratchet decelerates but does not stop.

Proof. (i) Wealth evolves as $W_{t+1} = Ak_t^{\alpha}\varepsilon_t - Rb_t - c_t$. After a sequence of bad shocks (ε_t low), wealth declines. Lower W_t reduces the buffer against future shocks, raising p. If this effect dominates the removal of high-σ households through default, p_t rises.

(ii) Default removes households with high realized default probability. If σ varies across households and high-σ households default first, the surviving pool has lower average σ and thus lower p. If this effect dominates wealth depletion, p_t falls. □

The empirical question is which effect dominates. For Bolivia, we expect negative duration dependence: the 2024–2026 crisis has already depleted household wealth, and continued macroeconomic stress will prevent wealth rebuilding. Each year of crisis lowers W_t further, raising p_t and accelerating the ratchet.

4.4 Correlated Shocks and Fire Sales

The aggregate shock η_t introduces correlation across households. When η_t is low (drought, price collapse, fuel shortage), many households experience negative income shocks simultaneously. This creates two amplification mechanisms.

First, correlated defaults. Let $D_t(\eta)$ denote defaults given aggregate shock η. When η is low, D_t is high:

$$D_t = \int_{\mathcal{U}_t} \mathbf{1}\{y_{it} + W_{it} < Rb_{it} + \underline{c}\} \, d\mu(i), \tag{38}$$

where $\mathcal{U}_t$ is the set of surviving unprotected households. The integral increases when η_t shifts all incomes downward.

Second, fire-sale price collapse. From the land price equation (7):

$$P_t^L = \bar{P}^L \cdot \left(\frac{\bar{D}}{D_t}\right)^{\theta}. \tag{39}$$

When D_t is high, P_t^L falls. Falling prices trigger additional defaults among households that could have sold land to repay debt at normal prices but cannot do so when prices crash.

Proposition 7 (Fire-Sale Amplification)**.** *Define the marginal defaulter as a household with debt b such that $P^L L = Rb$ under normal prices. When prices crash to $P_t^L < \bar{P}^L$:*

(i) Marginal defaulters cannot sell their way out of debt: $P_t^L L < Rb$.

(ii) They default even if their income shock was not severe.

(iii) The additional defaults raise D_t further, depressing prices more.

The feedback creates a death spiral: bad aggregate shock $\rightarrow$ correlated defaults $\rightarrow$ price crash $\rightarrow$ additional defaults $\rightarrow$ further price crash.

Proof. Let $b^* = P^L L/R$ be the debt level at which the household is exactly solvent under normal prices: selling land covers debt. For any household with $b > b^*$, forced sale at normal prices leaves a residual liability. For households with $b \leq b^*$, sale at normal prices allows debt repayment with remainder $P^L L - Rb \geq 0$.

When P_t^L falls to $\alpha \bar{P}^L$ (with $\alpha < 1$), the solvency threshold shifts to $b^{**} = \alpha P^L L/R < b^*$. Households with $b \in (b^{**}, b^*]$ could have avoided default at normal prices but cannot at fire-sale prices. These households enter D_t, raising it further and depressing P_t^L more. The process iterates until equilibrium. □

The fire-sale mechanism implies that ratchet dynamics are not smooth. In good years (high η_t), few households default, and land concentration proceeds slowly. In crisis years (low η_t), the fire-sale spiral generates mass defaults, and concentration jumps discretely. Bolivia's current macroeconomic crisis (depleted reserves, parallel exchange rate divergence, fuel shortages, negative GDP growth) is precisely the aggregate shock that triggers fire sales.

4.5 Half-Life of Smallholder Land

Define the half-life $t_{1/2}$ as the time until expected unprotected land falls to half its initial value:

$$\mathbb{E}[N_{t_{1/2}}^U] = \frac{N_0^U}{2}. \tag{40}$$

Proposition 8 (Half-Life Bounds). *(i) Under constant $p_t = \bar{p}$:*

$$t_{1/2} = \frac{\ln 2}{\bar{p}} \approx \frac{0.693}{\bar{p}}. \tag{41}$$

(ii) Under increasing p_t (negative duration dependence): $t_{1/2}$ is shorter.

(iii) Under decreasing p_t (positive duration dependence): $t_{1/2}$ is longer but finite.

Proof. (i) $\mathbb{E}[N_t^U] = N_0^U(1-\bar{p})^t$. Setting this equal to $N_0^U/2$: $(1-\bar{p})^{t_{1/2}} = 1/2$, so $t_{1/2} = \ln(2)/\ln(1/(1-\bar{p})) \approx \ln 2/\bar{p}$ for small $\bar{p}$.

(ii)–(iii) follow from the fact that $\prod_s(1-p_s)$ decays faster (slower) when p_s is increasing (decreasing). □

For calibration purposes: if the average annual default probability among converters is 5%, the half-life is approximately 14 years. If it is 10%, the half-life is 7 years. With adverse selection ($\mathbb{E}[p|\mathcal{C}] > \mathbb{E}[p]$) and negative duration dependence (crisis-driven wealth depletion), the effective $\bar{p}$ is higher than baseline estimates, and the half-life is correspondingly shorter.

4.6 Long-Run Distribution

Corollary 2 (Terminal Distribution). *As $t \to \infty$, the land distribution converges to:*

$$N_\infty^P = 1 - \lambda, \tag{42}$$

$$N_\infty^U = 0, \tag{43}$$

$$N_\infty^C = \lambda. \tag{44}$$

All converted land ends up in concentrated holdings. Protected land remains with original owners.

The long-run outcome has two classes of landowners: those who never converted (retaining N^P) and those who purchased foreclosed land (holding N^C). The original converters have disappeared from the land-owning population entirely.

Proposition 9 (Land Gini Coefficient). *Let G_t denote the land Gini coefficient at time t. Then:*

(i) $G_0 < G_t$ *for all* $t > 0$*: conversion and subsequent defaults increase inequality.*

(ii) G_t *is increasing in* t*: inequality rises monotonically.*

(iii) G_∞ *exceeds* G_0 *by an amount proportional to the conversion rate* λ *and the ratio of land values between concentrated and protected holdings.*

Proof. (i)–(ii): Each default transfers land from a smallholder (below-median holding) to a purchaser (above-median holding). The Lorenz curve shifts down with each transfer; the Gini rises.

(iii): The terminal Gini depends on the concentration of $N^C = \lambda$ among purchasers. If purchasers are few and large, the Gini increase is maximal. If purchasers are many and small, the Gini increase is smaller but still positive (since purchasers have above-median holdings by revealed preference—they have excess wealth to buy foreclosed land). □

4.7 Endogenous Entry

The baseline analysis assumed conversion occurs once at $t = 0$. In practice, Ley 1720 permits conversion at any time. This creates ongoing entry into the unprotected pool.

Proposition 10 (Ratchet with Endogenous Entry)**.** *Suppose households may convert at any* t*. Let* λ_t *denote the measure of new conversions at* t*. Then:*

$$N^U_{t+1} = N^U_t + \lambda_t - D_t. \tag{45}$$

The ratchet result holds if and only if $\sum_t \lambda_t < \infty$ *(finite total conversion) or* $\lim_t \lambda_t = 0$ *sufficiently fast that entry cannot replace exits.*

Proof. If new converters enter at rate λ_t and exit at rate $p_t N^U_t$, the steady state requires $\lambda_t = p_t N^U_t$. If λ_t decays faster than $p_t N^U_t$, the pool shrinks. Since N^U_t shrinks (by the ratchet), the required entry rate $\lambda^* = pN^U$ also shrinks. Unless new households willing to convert are inexhaustible, entry eventually falls below exit, and the ratchet proceeds. □

In Bolivia, the pool of potential converters is finite (440,000 households in protected-dominant regions). Once these households have either converted or decided not to, no new entrants remain. The ratchet then operates on the existing unprotected stock without replenishment.

5 Insurance

Protected tenure provides implicit insurance: land cannot be lost regardless of shocks. Conversion removes this insurance. The question is whether explicit insurance can substitute—and if so, how it changes selection and dynamics.

5.1 The Insurance Contract

An insurance contract specifies a premium π paid each period and a payout $I(\varepsilon)$ contingent on the shock. We focus on debt-service insurance: if $\varepsilon < \varepsilon^*(k, b)$, the insurer pays Rb on behalf of the household. The household avoids default but pays the premium regardless of whether a claim is made.

This contract replicates the key feature of protected tenure—land cannot be lost—while preserving credit access. The household bears income risk (bad shocks still reduce consumption) but not asset risk (land is never forfeited).

5.2 Actuarially Fair Premium

The actuarially fair premium equates expected payouts to premium revenue:

$$\pi^{\text{fair}}(A, W, \sigma; k, b) = p(A, W, \sigma; k, b) \cdot Rb, \tag{46}$$

where p is the default probability under the optimal uninsured policy.

The fair premium is household-specific. A household with high default probability (low A, high σ, high b/W) pays more. Implementing fair pricing requires the insurer to observe or infer (A, σ)—the same information problem that plagues credit markets.

Remark 1 (Premium as Signal)**.** *The fair premium reveals information. A household offered premium π learns that the insurer estimates its default probability at $p = \pi/Rb$. If the household* *believes its true p is lower, it may decline coverage. If it believes p is higher, it eagerly accepts. This is the standard adverse selection problem in insurance markets.*

5.3 Value Function with Insurance

Let $V^{U,I}$ denote the value of unprotected tenure with insurance:

$$V^{U,I}(\theta) = \max_{c,k,b} \left\{ u(c) + \beta \mathbb{E} \left[V^{U,I}(A, W', \sigma) \right] \right\} \tag{47}$$

subject to:

$$c + k + \pi = W + b, \tag{48}$$

$$b \leq \phi P^L, \tag{49}$$

$$W' = Ak^\alpha \varepsilon - Rb + I(\varepsilon), \tag{50}$$

where $I(\varepsilon) = Rb \cdot \mathbf{1}\{\varepsilon < \varepsilon^*\}$.

The key difference from uninsured tenure: the transition for W' includes the insurance payout when the shock is bad. This ensures $W' \geq Ak^\alpha \varepsilon - Rb + Rb = Ak^\alpha \varepsilon \geq 0$ even in default states, preventing land loss.

Proposition 11 (No Default with Insurance)**.** *Under full debt-service insurance with actuarially fair pricing, no household defaults. The default indicator* $\mathbf{1}_D = 0$ *with probability one.*

Proof. Default occurs when $W' < \underline{c}$, i.e., $Ak^\alpha \varepsilon - Rb < \underline{c}$. With insurance, the payout $I = Rb$ when $\varepsilon < \varepsilon^*$, so $W' = Ak^\alpha \varepsilon \geq 0$. The household may have low consumption but retains its land. □

5.4 Insurance Changes Selection

With insurance available, the conversion decision becomes a comparison among three options:

1. **Stay protected**: Value V^P. No credit access, no premium, no land loss risk.
2. **Convert with insurance**: Value $V^{U,I}$. Credit access, premium cost, no land loss risk.

3. **Convert without insurance**: Value V^U. Credit access, no premium, land loss risk.

Under mandatory insurance, option 3 is unavailable. The conversion decision reduces to $V^{U,I} \gtrless V^P$.

Theorem 4 (Insurance Filters Selection). *If insurance is mandatory as a condition of conversion:*

(i) The conversion threshold rises: $\bar{A}^I(W, \sigma) > \bar{A}(W, \sigma)$ *for all* (W, σ).

(ii) Fewer households convert: $\lambda^I < \lambda$.

(iii) The converting population has lower average default probability: $\mathbb{E}[p|\mathcal{C}^I] < \mathbb{E}[p|\mathcal{C}]$.

(iv) The ratchet stops: $D_t = 0$ *for all* t *(no defaults among insured converters).*

Proof. (i): The conversion gain with insurance is:

$$\Delta^I = V^{U,I} - V^P = (V^{U,I} - V^U) + (V^U - V^P) = -\text{premium cost} + \Delta. \tag{51}$$

Since the premium is positive ($\pi > 0$), $\Delta^I < \Delta$. A household that was indifferent without insurance ($\Delta = 0$) strictly prefers protection with insurance ($\Delta^I < 0$). The threshold $\bar{A}^I$ solving $\Delta^I = 0$ exceeds $\bar{A}$.

(ii): Higher threshold means smaller conversion region: $\mathcal{C}^I \subset \mathcal{C}$, so $\lambda^I < \lambda$.

(iii): The households excluded by insurance—those in $\mathcal{C} \setminus \mathcal{C}^I$—are precisely those for whom the premium exceeds the value of credit access. These are high-p households (since $\pi \propto p$). Removing them lowers $\mathbb{E}[p|\mathcal{C}^I]$.

(iv): By Proposition 11, insured households do not default. With $D_t = 0$, the ratchet halts. □

The theorem establishes that mandatory insurance solves the adverse selection problem by changing who converts. The premium acts as a screening device: households with high default probability face high premiums, making conversion unattractive. They remain protected—the efficient outcome for them—rather than converting, borrowing, defaulting, and losing their land.

5.5 The Premium as Screening Device

Proposition 12 (Screening Properties)**.** *The actuarially fair premium $\pi^{fair}(\theta) = p(\theta) \cdot Rb$ screens households as follows:*

(i) Households with $p > p^*$ face premiums exceeding their credit benefit and do not convert.

(ii) Households with $p < p^*$ face affordable premiums and convert.

(iii) The threshold p^* is endogenous, determined by the credit benefit schedule $B(A, W)$.

Proof. A household converts with insurance if $V^{U,I} > V^P$, i.e., if:

$$\underbrace{V^U - V^P}_{\text{gross benefit } \Delta} > \underbrace{V^U - V^{U,I}}_{\text{premium cost}}. \tag{52}$$

The premium cost scales with p: $V^U - V^{U,I} \approx \sum_t \beta^t \pi = \sum_t \beta^t p \cdot Rb$. If p is high, the premium cost exceeds Δ, and the household does not convert. The threshold p^* solves $\Delta(A, W, \sigma) = \text{premium cost}(p)$. $\square$

This screening is precisely what Ley 1720 lacks. Without mandatory insurance, all households with $\Delta > 0$ convert—including the high-p "desperate" households identified in Theorem 2. With mandatory insurance, these households are screened out.

5.6 Voluntary versus Mandatory Insurance

If insurance is voluntary, households choose among all three options. Which households decline coverage?

Proposition 13 (Selection into Uninsurance)**.** *When insurance is voluntary, households that convert without insurance are:*

(i) Those who underestimate their default probability: $\hat{p} < p$.

(ii) Those who are present-biased: high weight on immediate credit benefit, low weight on future default risk.

(iii) Those who cannot afford the premium: $\pi > W - \underline{c}$ (liquidity-constrained).

In all cases, the uninsured pool has higher realized default rates than the insured pool.

This is adverse selection within the insurance market, compounding the adverse selection into conversion. Households that decline coverage are those most likely to default. The uninsured pool generates defaults; the insured pool does not. The ratchet operates on the uninsured segment.

Corollary 3 (Mandatory Dominates Voluntary)**.** *Mandatory insurance as a conversion requirement dominates voluntary insurance:*

(i) It eliminates adverse selection in the insurance market.

(ii) It screens out non-viable conversions.

(iii) It reduces defaults to zero and halts the ratchet.

(iv) It improves welfare for infra-marginal converters (who pay lower premiums in the absence of adverse selection).

5.7 Welfare Comparison

Define ex-ante welfare as expected lifetime utility at $t = 0$:

$$\mathcal{W}^{(j)} = \int \max\{V^P, V^{(j)}\}\, dF(\theta), \tag{53}$$

where $j \in \{\text{no conversion}, U, U + \text{vol.ins}, U + \text{mand.ins}\}$.

Proposition 14 (Ex-Ante Welfare Ordering)**.** *Evaluated at $t = 0$:*

$$\mathcal{W}^{\textit{no conversion}} < \mathcal{W}^{\textit{mand.ins}} < \mathcal{W}^{\textit{vol.ins}} < \mathcal{W}^{\textit{no ins}}. \tag{54}$$

The ordering appears paradoxical: the no-insurance regime has the highest ex-ante welfare. But this reflects option value, not outcomes. With no insurance, households have access to the high-variance gamble (convert, borrow, possibly default). Some will win; many will lose. The expected value of the gamble—evaluated before outcomes are realized—exceeds the certain value of protection or the insurance-constrained value of conversion.

Proposition 15 (Ex-Post Welfare Reversal)**.** *Evaluated at $t = T$ for large T:*

(i) The no-insurance regime has the highest inequality (maximal land Gini).

(ii) The mandatory-insurance regime has the lowest inequality (Gini $\approx G_0$).

(iii) Under any inequality-averse social welfare function, the ordering reverses:

$$\mathcal{W}_T^{no\ ins} < \mathcal{W}_T^{vol.ins} < \mathcal{W}_T^{mand.ins}. \tag{55}$$

Proof. (i)–(ii): Follow from the ratchet analysis. Defaults transfer land from smallholders to purchasers. More defaults (no insurance) mean more transfers and higher Gini.

(iii): An inequality-averse SWF penalizes the variance in outcomes. The no-insurance regime has high variance (some households lose everything); mandatory insurance has low variance (no household loses land). The penalty dominates the option-value gain. □

The welfare reversal clarifies the policy choice. A utilitarian planner maximizing ex-ante expected utility prefers no insurance. A Rawlsian planner maximizing the minimum outcome prefers mandatory insurance. The constitutional framers of Article 394-II—who declared pequeña propiedad inalienable—revealed a Rawlsian preference. Ley 1720 reverses this without changing the social welfare function.

5.8 Information Requirements

Fair pricing requires observing (A, σ) or proxies. In practice, insurers use:

- **Location**: agroecological zone, soil type, historical weather.
- **Crop mix**: rainfed vs. irrigated, subsistence vs. commercial.
- **Credit history**: past loans, repayment record.
- **Land value**: cadastral assessment.

Bolivia's existing agricultural insurance program—Seguro Agrario Pachamama—covers weather risk but not credit risk. It is indexed (payouts based on weather station data, not individual losses) and subsidized (premiums below actuarial cost). Extending this infrastructure to cover debt-service risk would require:

1. Linking insurance to credit contracts (mandatory purchase at loan origination).
2. Pricing by loan-to-value ratio and borrower characteristics.

3. Reinsurance for correlated risk (drought affecting many borrowers simultaneously).

5.9 The Ratchet with Insurance

Corollary 4 (Ratchet Halts)**.** *Under mandatory insurance:*

$$D_t = 0 \quad \forall t, \qquad N_t^U = N_0^{U,I} \quad \forall t, \qquad N_t^C = 0 \quad \forall t. \tag{56}$$

The ratchet does not operate. All converters retain their land indefinitely.

The corollary understates the benefit. With no defaults, there are no fire sales. Land prices remain stable. Wealth is not transferred from smallholders to purchasers. The long-run distribution equals the initial distribution (adjusted for the smaller conversion rate under insurance).

5.10 Residual Risk

Insurance does not eliminate all risk. Three residual risks remain:

First, aggregate shocks exceeding insurance capacity. If a drought affects all insured households simultaneously, the insurer may be unable to pay all claims. This is reinsurance risk, requiring either public backstop or private reinsurance markets.

Second, basis risk in indexed insurance. If payouts depend on weather station data rather than individual losses, a household may experience a bad outcome (localized pest infestation) without triggering a payout. This risk exists under Seguro Agrario Pachamama and would persist under debt-service insurance.

Third, premium affordability. Households must pay the premium out of current income. In bad years, premium payment may force consumption below subsistence, defeating the purpose of insurance. Premium subsidies or flexible payment schedules address this.

These residual risks are second-order relative to the uninsured baseline. The primary risk—losing land through foreclosure—is eliminated. The secondary risks can be managed through program design.

6 Labor Reallocation and Rental Markets

The baseline model treats conversion as a credit-access decision. Households convert to borrow, invest, and raise productivity. But the empirical literature on land reform identifies a different primary channel. **?**, studying Mexico's 1992 ejido reform—the closest parallel to Ley 1720—find that welfare gains came primarily through labor reallocation and migration, not credit. The rental market, which allows households to lease land while retaining ownership, enabled participation in off-farm labor markets without irreversible land loss.

This section extends the model to include occupational choice, rental markets, and permanent exit. The extension yields three results: (i) the rental channel provides gains from conversion separate from credit access; (ii) rental partially substitutes for the insurance value of inalienability; (iii) adverse selection is mitigated when high-risk households sort into rental rather than credit.

6.1 Extended Environment

Off-Farm Labor. An off-farm labor market exists with wage $w^O > 0$. Access requires either permanent migration (selling land and exiting agriculture) or temporary migration (renting out land and working off-farm while retaining ownership).

Rental Market. Under unprotected tenure, land can be rented at rate $r^L > 0$ per hectare per year. The rental market is competitive; the rate clears supply (households renting out) and demand (commercial farmers seeking additional land). Under protected tenure, rental is prohibited—inalienability covers use rights as well as ownership.

Land Sales. Under unprotected tenure, land can be sold at price P^L. The seller receives a lump sum and exits agriculture permanently. Under protected tenure, sale is prohibited.

6.2 Choice Set Expansion

Under **protected tenure**, the household chooses between:

(P1) **Farm**: Operate the farm with self-financed capital. Value $V^{P,\text{farm}}(A, W)$.

(P2) **Forfeit and exit**: Abandon land (receiving nothing), work at wage w^O forever. Value $V^{\text{exit}} = u(w^O)/(1-\beta)$.

Under protected tenure, the household cannot sell land, so exit means forfeiting it entirely. This makes exit unattractive unless farming productivity is extremely low.

Under **unprotected tenure**, the household chooses among:

(U1) **Farm with credit**: Borrow up to ϕP^L, invest, face default risk. Value $V^{U,\text{farm}}(A, W, \sigma)$.

(U2) **Rent out and work off-farm**: Lease land at rate r^L, earn wage w^O, retain ownership. Income $w^O + r^L L$ is deterministic. Value $V^{U,\text{rent}}(W)$.

(U3) **Sell and exit**: Sell land at price P^L, receive lump sum, work at wage w^O forever. Value $V^{U,\text{sell}}(W)$.

The value function under unprotected tenure becomes:

$$V^U(A, W, \sigma) = \max\left\{V^{U,\text{farm}}(A, W, \sigma),\ V^{U,\text{rent}}(W),\ V^{U,\text{sell}}(W)\right\}. \tag{57}$$

6.3 Value Functions

Farming with credit. As in the baseline model:

$$V^{U,\text{farm}}(A, W, \sigma) = \max_{k,b}\left\{u(c) + \beta\left[(1-p)\mathbb{E}[V^U|\text{survive}] + p \cdot V^L\right]\right\}, \tag{58}$$

with default probability $p(A, W, \sigma; k, b)$.

Renting out. The household earns deterministic income $w^O + r^L L$:

$$V^{U,\text{rent}}(W) = \max_c\left\{u(c) + \beta V^U(A, W', \sigma)\right\}, \tag{59}$$

subject to $c + W' = W + w^O + r^L L$. The household retains ownership and the option to farm next period. There is no production risk and no default risk (no debt).

Selling and exiting. The household receives the land value and exits:

$$V^{U,\text{sell}}(W) = u(W + P^L L + w^O) + \beta V^{\text{exit}}, \tag{60}$$

where $V^{\text{exit}} = u(w^O)/(1-\beta)$. This is a terminal decision: land is gone, and the household works at wage w^O indefinitely.

6.4 Rental as Partial Insurance

The rental option provides a floor on utility that does not depend on farming productivity or production risk.

Proposition 16 (Rental Floor)**.** *For any household under unprotected tenure:*

$$V^U(A, W, \sigma) \geq V^{U,rent}(W) > V^L. \tag{61}$$

The rental option bounds the value function from below. No household need default if rental income covers subsistence.

Proof. The rental option is always available and yields deterministic income $w^O + r^L L$. The household can always choose it, so $V^U \geq V^{U,\text{rent}}$. If $w^O + r^L L \geq \underline{c}$, rental income covers subsistence, making it preferred to default (which yields V^L). □

The proposition implies that rental substitutes, partially, for the insurance function of inalienability. A household facing a bad shock can switch to rental, avoiding default. The option value of this switch is positive even for households that farm in equilibrium.

Corollary 5 (Lower Default Rate)**.** *With the rental option, the default rate among unprotected households is lower than in the baseline model:*

$$\bar{p}^{rental} < \bar{p}^{baseline}. \tag{62}$$

Proof. Households that would default under farming-with-credit may instead choose rental, which has zero default probability. The aggregate default rate falls. □

6.5 Decomposing Conversion Gains

The conversion gain now has three components:

Proposition 17 (Three Channels)**.** *The conversion gain decomposes as:*

$$\Delta(A, W, \sigma) = \Delta^{credit} + \Delta^{rental} + \Delta^{sale}, \tag{63}$$

where:

$$\Delta^{credit} = V^{U,farm} - V^{P,farm}, \tag{64}$$

$$\Delta^{rental} = \max\{V^{U,rent} - V^{P,farm}, 0\}, \tag{65}$$

$$\Delta^{sale} = \max\{V^{U,sell} - V^{P,farm}, 0\}. \tag{66}$$

The baseline model sets $\Delta^{\text{rental}} = \Delta^{\text{sale}} = 0$. The extended model shows these can be first-order—and for some households, the rental or sale channel dominates credit.

6.6 Channel Sorting

Proposition 18 (Who Uses Each Channel)**.** *Households sort into channels based on (A, W, σ):*

(i) ***High-**A: Farm with credit. The productivity gain from investment dominates rental income.*

(ii) ***Low-**A**, moderate-**W: Rent out and work off-farm. Low productivity makes farming unattractive; $w^O + r^L L$ exceeds expected farming income.*

(iii) ***Low-**A**, high-**W: Sell and exit. The land value $P^L L$ is large; farming income is small; exit is optimal.*

(iv) ***High-**σ: Rental is attractive because it avoids production risk. High-σ households shift from credit to rental relative to baseline.*

(v) ***Low-**A**, low-**W: May farm with credit if desperate for liquidity (the desperation mechanism), or rent if $w^O + r^L L \geq \underline{c}$.*

Proof. Comparison of the three value functions. $V^{U,\text{farm}}$ is increasing in A (higher marginal product) and decreasing in σ (higher default risk). $V^{U,\text{rent}}$ is independent of A and σ.

$V^{U,\text{sell}}$ is independent of A and σ but increasing in P^L. The household chooses the highest value. □

The sorting result implies that the composition of converters depends on the relative magnitudes of (A, w^O, r^L, P^L). If off-farm wages are high (w^O large), the rental channel attracts more households. If land prices are high (P^L large), the sale channel attracts more.

6.7 Rental Mitigates Adverse Selection

Theorem 5 (Reduced Adverse Selection)**.** *With the rental option, adverse selection into conversion is weaker:*

$$\mathbb{E}[p|\mathcal{C}]^{rental} < \mathbb{E}[p|\mathcal{C}]^{baseline}. \tag{67}$$

Proof. Partition converters by channel:

$$\mathcal{C} = \mathcal{C}^{\text{farm}} \cup \mathcal{C}^{\text{rent}} \cup \mathcal{C}^{\text{sell}}. \tag{68}$$

Households in $\mathcal{C}^{\text{rent}}$ and $\mathcal{C}^{\text{sell}}$ have $p = 0$: they do not borrow, so they cannot default. Only households in $\mathcal{C}^{\text{farm}}$ have $p > 0$.

The average default probability among converters is:

$$\mathbb{E}[p|\mathcal{C}] = \frac{|\mathcal{C}^{\text{farm}}|}{|\mathcal{C}|} \cdot \mathbb{E}[p|\mathcal{C}^{\text{farm}}]. \tag{69}$$

In the baseline model, $\mathcal{C}^{\text{rent}} = \mathcal{C}^{\text{sell}} = \emptyset$. All converters are in $\mathcal{C}^{\text{farm}}$, so $\mathbb{E}[p|\mathcal{C}] = \mathbb{E}[p|\mathcal{C}^{\text{farm}}]$.

With rental, some households shift from $\mathcal{C}^{\text{farm}}$ to $\mathcal{C}^{\text{rent}}$. The weight on $\mathbb{E}[p|\mathcal{C}^{\text{farm}}]$ falls, reducing $\mathbb{E}[p|\mathcal{C}]$.

Moreover, the households that shift are those with high σ (for whom default risk makes farming unattractive) and low A (for whom rental income exceeds farming income). These are high-p households. Their exit from $\mathcal{C}^{\text{farm}}$ also reduces $\mathbb{E}[p|\mathcal{C}^{\text{farm}}]$ itself. □

The theorem identifies the mechanism through which rental markets improve outcomes: they provide an alternative to credit-financed farming for households that would otherwise convert, borrow, and default. The ratchet operates only on $\mathcal{C}^{\text{farm}}$; households in $\mathcal{C}^{\text{rent}}$ retain their land.

6.8 The Ratchet with Rental

Corollary 6 (Slower Ratchet)**.** *With the rental option, the ratchet half-life is longer:*

$$t_{1/2}^{rental} > t_{1/2}^{baseline}. \tag{70}$$

Proof. The half-life is $t_{1/2} \approx \ln 2/\bar{p}$. With $\bar{p}^{\text{rental}} < \bar{p}^{\text{baseline}}$ (Corollary 5), the half-life increases. □

The ratchet does not stop—households in $\mathcal{C}^{\text{farm}}$ still face default risk—but it slows. Land concentration proceeds at a lower rate.

6.9 Sale Channel and Land Concentration

The sale channel ($\mathcal{C}^{\text{sell}}$) differs from rental in a critical respect: it permanently transfers land from smallholders to purchasers.

Proposition 19 (Sale Feeds the Ratchet)**.** *Sales of land by converting households contribute to land concentration even without default:*

$$N_t^C = D_t + S_t, \tag{71}$$

where S_t is the measure of land sold (not foreclosed) at time t.

The distinction matters for welfare. A household that sells land at price P^L receives fair value; a household that defaults loses land at fire-sale prices and bears the utility cost of landlessness. Both contribute to concentration, but the welfare implications differ.

6.10 What Bolivia's Law Is Missing

Ley 1720 permits conversion but does not create a rental market. Under current law:

- *Pequeña propiedad* cannot be rented (inalienability covers use rights).
- *Mediana propiedad* can be rented, but no formal rental infrastructure exists.
- Households converting under Ley 1720 gain the sale and credit channels, but the rental channel remains underdeveloped.

The de Janvry et al. finding suggests that rental, not credit, was the primary channel for welfare gains in Mexico. If Bolivia follows this pattern, the law addresses the wrong margin. Households gain access to credit (high-risk, high-variance) but limited access to rental (low-risk, low-variance).

Remark 2 (Policy Implication)**.** *Developing formal rental markets—contract enforcement, standard lease terms, registration—would provide gains from conversion that do not require bearing credit risk. For households with low farming productivity, renting to commercial operators while working off-farm dominates both protected tenure (no rental income) and credit-financed farming (default risk). The policy priority should be rental infrastructure alongside or instead of credit access.*

6.11 Calibration Considerations

To calibrate the extended model, we require:

- **Off-farm wage** w^O: From labor force surveys, the median urban wage for rural-to-urban migrants. For Bolivia, approximately Bs 2,500/month (informal sector).
- **Rental rate** r^L: From agricultural surveys, the average land rental rate. Highly variable by region: Bs 500–1,500/hectare/year.
- **Land price** P^L: From INRA cadastral data. Ranges from Bs 15,000/hectare (Altiplano) to Bs 50,000/hectare (Lowlands).

The key ratio is $r^L L/(Ak^\alpha)$: when rental income exceeds expected farming income, the rental channel dominates. For low-A households—precisely those at risk of default under the credit channel—rental is often the superior option.

6.12 Summary

The labor reallocation and rental extension yields three main results:

First, the rental channel provides gains from conversion separate from credit. Households can participate in off-farm labor markets while retaining land ownership and bearing no default risk.

Second, rental partially substitutes for the insurance value of inalienability. The rental option bounds utility from below, preventing default for households with low farming productivity.

Third, adverse selection is mitigated. High-risk households sort into rental rather than credit, reducing the average default probability among converters and slowing the ratchet.

Bolivia's Ley 1720 activates the credit and sale channels but not the rental channel. If de Janvry et al.'s findings generalize, the law addresses the less important margin while missing the primary source of welfare gains.

7 Calibration

This section calibrates the model to Bolivian microdata and generates quantitative predictions. The calibration draws on two sources: the 2013 National Agricultural Census (CNA-2013), which provides UPA-level data on land tenure, credit access, and capital for 871,927 agricultural production units; and the 2015 Agricultural Survey (EA-2015), which provides output values for a stratified sample weighted to 628,000 UPAs. The key empirical findings are: (i) Bolivia's land Gini is 0.914—among the highest in Latin America—with 0.4% of UPAs controlling 47% of agricultural land; (ii) the collateral constraint mechanism operates as the De Soto hypothesis predicts; (iii) smallholders have higher per-hectare productivity than large commercial farms.

7.1 Data

The CNA-2013 recorded 871,927 agricultural production units (UPAs) across Bolivia's nine departments. The census collected information on land tenure (Q21), credit application and outcomes (Q108–Q112), equipment ownership, livestock, and household demographics. We extract UPA-level microdata via the REDATAM system.

We define **protected tenure** as UPAs reporting any of: *propiedad cedida por la comunidad* (community-granted property), participation in traditional labor exchange (*minka/ayni*), or legal status as *comunidad*. This proxy captures the 61.3% of UPAs operating under tenure arrangements that share the key feature of *pequeña propiedad*: inalienability and protection from seizure.

The EA-2015 is used for output-based statistics ($\sigma_{\log y}$, productivity by tenure) because the CNA-2013 does not record crop prices at the UPA level.

7.2 Land Distribution

Bolivia has among the highest land inequality in Latin America. Table 1 presents the distribution of land across size classes.

Table 1: Land Distribution by Size Class (CNA-2013)

Size class	N UPAs	% UPAs	Total ha	% ha	Mean ha	Median ha
0–5 ha (minifundio)	517,553	59.4	738,610	2.1	1.4	1.0
5–20 ha (pequeña)	211,083	24.2	2,025,571	5.8	9.6	9.0
20–50 ha (mediana pequeña)	69,402	8.0	2,048,759	5.9	29.5	27.0
50–200 ha (mediana)	52,778	6.1	4,006,463	11.6	75.9	60.0
200–500 ha (empresa pequeña)	9,833	1.1	3,017,517	8.7	306.9	300.0
500–2000 ha (empresa mediana)	7,823	0.9	6,496,424	18.7	830.4	640.0
>2000 ha (empresa grande)	3,455	**0.4**	16,321,640	**47.1**	4,724	3,400
Total	**871,927**	100	**34,654,984**	100	39.7	2.5

Notes: UPA = Unidad de Producción Agropecuaria (agricultural production unit). Hectares summed across Q21 tenure categories. Source: CNA-2013 UPA-level microdata.

The distribution is bimodal: 59% of UPAs (minifundios under 5 hectares) share 2% of the land, while 0.4% of UPAs (holdings over 2,000 hectares) control 47%. The median

UPA is 2.5 hectares; the mean is 39.7 hectares—a ratio of 16:1 indicating extreme right-skewness.

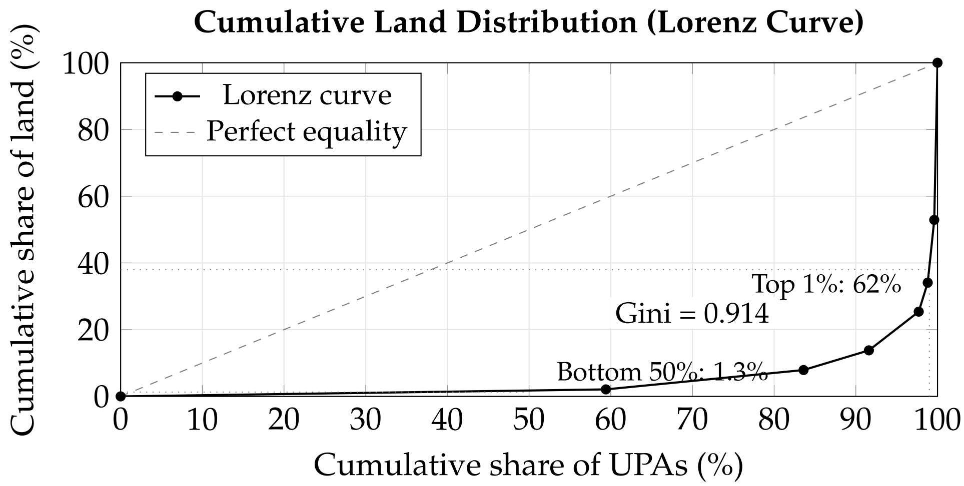

Figure 2: Lorenz curve for land distribution. The land Gini coefficient, calculated from 871,927 UPA-level observations, is 0.914. The bottom 50% of UPAs hold 1.3% of agricultural land; the top 1% hold 62%. Source: CNA-2013.

The land Gini coefficient, computed from the full distribution of 871,927 UPA-level hectares, is **0.914**—comparable to Chile (0.91, FAO 1997) and Guatemala (≈0.93), and higher than Brazil (0.85, FAO 1996) or Colombia (0.80, FAO 2001). Bolivia sits at the high end of the Latin American range.

7.3 Summary Statistics: Credit Access

Table 2 presents credit access and the collateral constraint.

Three patterns emerge. First, credit application is low: only 10.9% of UPAs sought credit. This may reflect self-selection (households anticipating rejection do not apply) or low demand. Second, among applicants, the rejection rate is 23%—higher than typical for agricultural credit in Latin America. Third, among those rejected, nearly one-quarter cite lack of collateral as the reason. This is the De Soto mechanism in action.

Table 2: Credit Access and Collateral Constraints (CNA-2013)

Statistic	Value
Sample	
Total UPAs	871,927
Protected tenure (proxy)	61.3%
Credit access	
Applied for credit	10.9%
Obtained credit	8.4%
Rejection rate (among applicants)	23.0%
Collateral constraint	
Rejected for "falta de garantía"	23.1% of rejections
(share of all UPAs)	0.58%
Capital and insurance	
Uses tractor (own or rented)	38.2%
Has agricultural insurance	3.1%
Receives technical assistance	16.9%

7.4 The De Soto Mechanism: Descriptive Evidence

Table 3 disaggregates credit outcomes by tenure type. The contrast between protected and unprotected tenure is stark.

Table 3: Credit Outcomes by Tenure Type

Group	N (000s)	Applied (%)	Obtained (%)	Rejected (%)	Garantía (%)
By tenure					
Has propiedad only	757	10.7	8.3	22.3	22.9
Has comunidad (cedida)	84	12.7	9.1	28.8	24.4
Protected (any)	534	11.0	8.5	22.5	26.0
Unprotected (empresarial)	0.65	32.3	28.8	11.0	39.1
By capital					
Uses tractor	333	14.5	12.0	17.2	26.2
Has cattle	322	11.8	9.2	21.8	24.0

Notes: "Rejected" is rejection rate among applicants. "Garantía" is the share of rejections citing lack of collateral. Sample: CNA-2013.

The "Legal Empresarial" category (N = 650 UPAs with corporate/commercial registration) shows the unprotected benchmark: 32.3% apply for credit, 28.8% obtain it, and only 11% are rejected. By contrast, protected-tenure UPAs apply at one-third the rate (11%), face double the rejection rate (22.5%), and—crucially—cite collateral constraints more frequently (26.0% vs. 39.1%). The higher rate among empresarial may reflect larger loan requests hitting binding collateral limits; the protected rate reflects the inalienability constraint directly.

7.5 The De Soto Mechanism: Regression Evidence

To isolate the tenure effect from confounders (location, scale, capital), we estimate three regressions. The key outcome is whether a rejected applicant cites "falta de garantía" as the rejection reason.

Specification. The estimating equation is:

$$Y_i = \alpha + \beta \cdot \text{Protected}_i + X_i' \gamma + \delta_d + \varepsilon_i, \tag{72}$$

where Y_i is the outcome (rejection, collateral rejection), Protected_i is the tenure proxy, X_i includes controls (log hectares, tractor use, cattle, education), and δ_d are department fixed effects. Standard errors are heteroskedasticity-robust.

Table 4 presents results for three specifications.

Column (1) shows that protected-tenure proxies have mixed effects on rejection probability: households with comunidad tenure are 1.8pp more likely to be rejected, but those using traditional labor exchange (comunitario) are 4.7pp *less* likely. The latter may reflect stronger community support networks.

Column (2) is the key specification. Among rejected applicants, those with comunidad tenure are **3.6 percentage points** more likely to cite lack of collateral as the rejection reason ($p < 0.001$). Participation in comunitario labor exchange raises this by **7.1 percentage points**. These are large effects: the baseline rate is 23%, so the protected-tenure premium represents a 15–30% increase in collateral-constraint salience.

Column (3) shows the unconditional effect (on all UPAs). The coefficients are tiny because only 0.58% of all UPAs are rejected for lack of collateral. But the tenure effect

Table 4: The Collateral Constraint: Regression Evidence

	(1) Rejected | Applied	(2) Garantía | Rejected	(3) Garantía (Unconditional)
Has comunidad	0.018***	0.036***	0.001***
	(0.005)	(0.010)	(0.000)
Comunitario labor	−0.047***	0.071***	0.001***
	(0.003)	(0.006)	(0.000)
Log hectares	−0.011***	0.019***	0.001***
	(0.001)	(0.001)	(0.000)
Uses tractor	−0.094***	0.029***	0.001***
	(0.003)	(0.006)	(0.000)
Department FE	Yes	Yes	Yes
Controls	Yes	Yes	Yes
N	95,384	21,971	871,927
R^2	0.107	0.026	0.001

Notes: OLS with robust standard errors in parentheses. Column (1): sample is applicants; outcome is rejection indicator. Column (2): sample is rejected applicants; outcome is "falta de garantía" indicator. Column (3): full sample; outcome is "falta de garantía" indicator. Controls: has_cattle, tech_assistance, has_insurance, max_educ, max_age. *** p<0.01, ** p<0.05, * p<0.1.

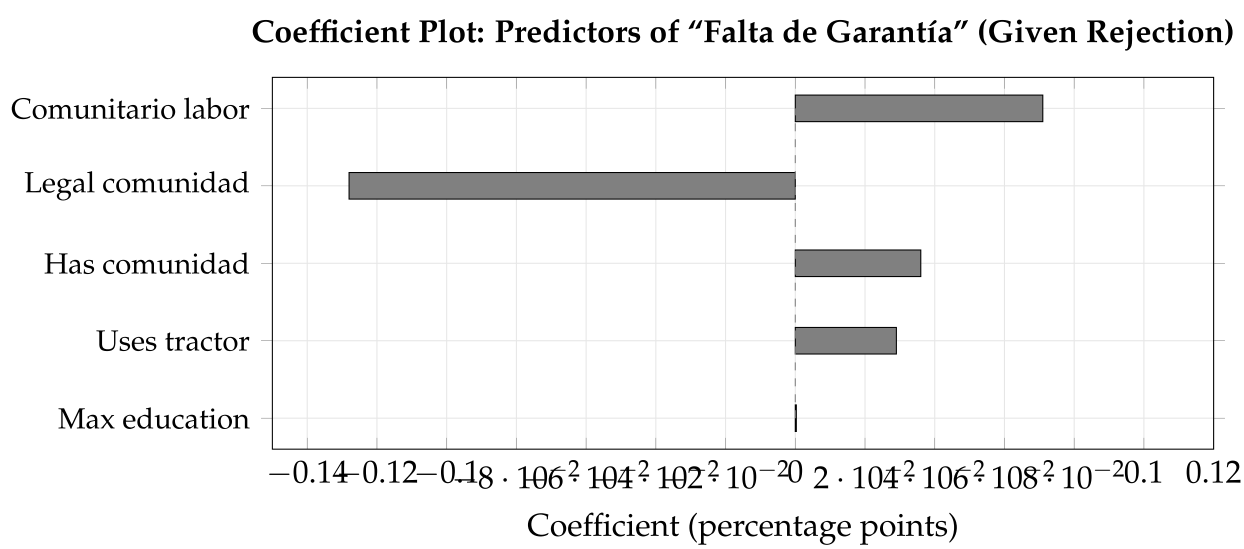

Figure 3: Coefficient plot for the key regression: probability of citing "falta de garantía" as rejection reason, conditional on being rejected. Bars show point estimates. Protected-tenure proxies ("Has comunidad," "Comunitario labor") significantly increase the probability of citing collateral constraints. "Legal comunidad" shows a negative effect but with a wide confidence interval due to small sample (N = 683 legal comunidad UPAs). Data: CNA-2013, N = 21,971 rejected applicants.

remains statistically significant: protected tenure raises the probability by 0.1 percentage points.

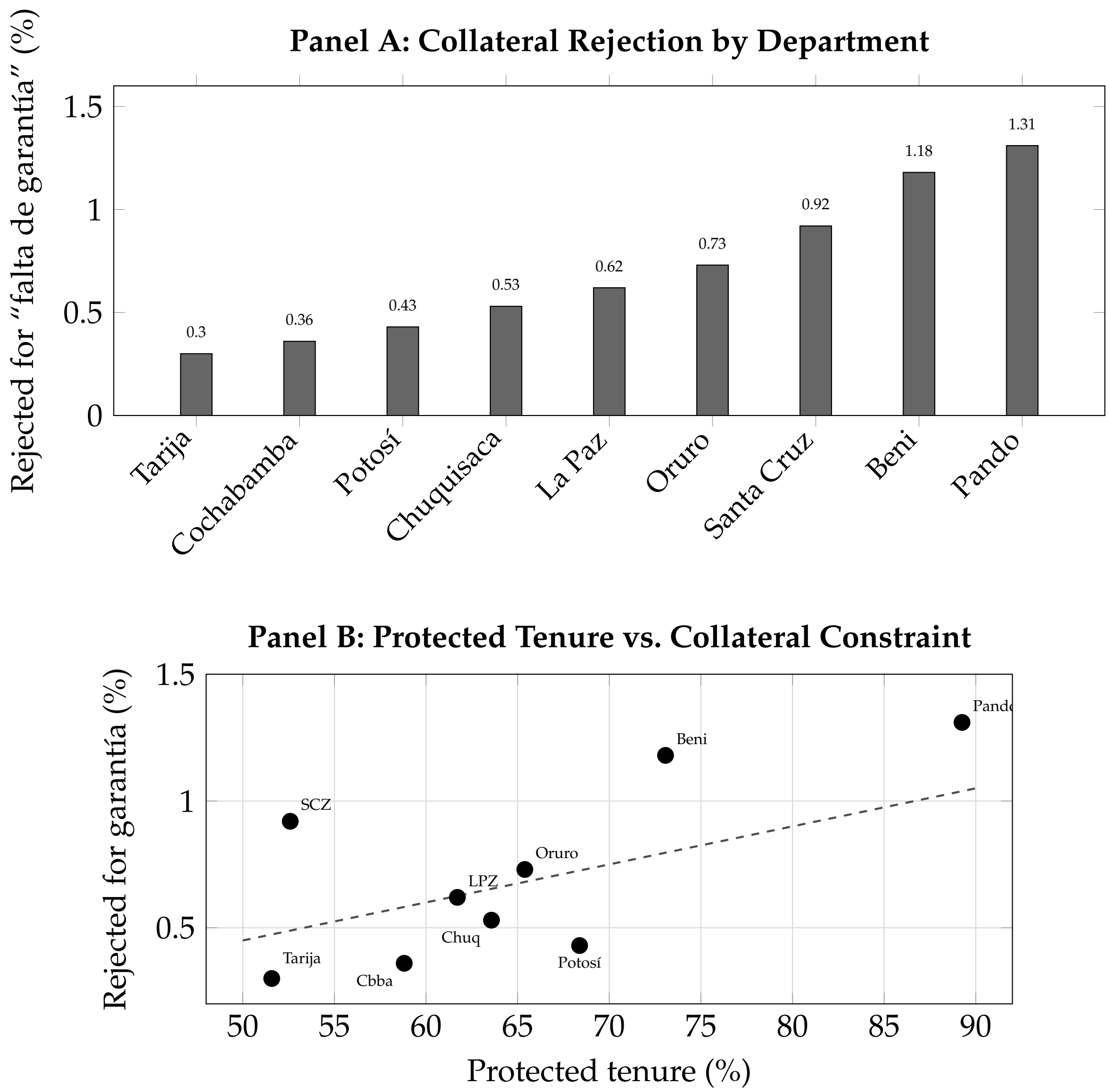

Figure 4: Geographic variation in the collateral constraint. Panel A orders departments by rejection rate for "falta de garantía." Panel B shows the cross-sectional correlation between protected tenure share and collateral-rejection rate. The regression line is illustrative. Data: CNA-2013, N = 871,927 UPAs.

7.6 The Credit-Capital Link

The De Soto hypothesis predicts that collateral constraints reduce capital investment. Table 5 tests this by regressing tractor use on credit access and tenure.

Table 5: Tractor Use and Credit Access

	Uses Tractor
Credit obtained	0.135***
	(0.002)
Has comunidad	−0.128***
	(0.002)
Legal empresarial	0.141***
	(0.018)
Log hectares	0.018***
	(0.000)
Has cattle	0.168***
	(0.001)
Department FE	Yes
N	871,927
R^2	0.105

Notes: OLS with robust standard errors. Outcome is indicator for tractor use (owned or rented). *** p<0.01.

Obtaining credit raises tractor use by **13.5 percentage points**—a 35% increase relative to the 38% baseline. Comunidad tenure *reduces* tractor use by 12.8pp, while empresarial tenure *raises* it by 14.1pp. The pattern is consistent with the model: protected tenure limits credit access; credit enables capital acquisition; capital raises productivity.

7.7 Production Function

The Cobb-Douglas production function is estimated from CNA-2013 community-level aggregates (N = 25,333 communities):

$$\log(VBP) = \alpha_L \cdot \log(\text{ha}) + \alpha_K \cdot \log(\text{tractors}/1000) + \alpha_{TLU} \cdot \log(1 + TLU) + \gamma \cdot \text{irrigated} + \delta_d + \varepsilon \tag{73}$$

The capital share $\alpha_K = 0.246$ implies moderate returns to capital investment. Returns to scale are 0.77—decreasing returns, consistent with smallholder agriculture.

Table 6: Production Function Estimates

Parameter	Estimate	SE
α_L (land)	**0.335**	(0.007)
α_K (capital)	**0.246**	(0.005)
α_{TLU} (livestock)	0.185	(0.009)
Irrigation premium	0.347	(0.025)
Returns to scale ($\alpha_L + \alpha_K + \alpha_{TLU}$)	**0.766**	
N (communities)	25,333	
R^2	0.632	

Notes: OLS with HC3 robust standard errors. Department fixed effects included. Source: CNA-2013 community-level aggregates.

7.8 Output Variance

The output variance $\sigma_{\log y}$ is computed from EA-2015 UPA-level VBP (valor bruto de producción). Protected-tenure UPAs have lower variance ($\sigma = 1.655$) than empresarial ($\sigma = 2.076$), consistent with the model's assumption that protected tenure provides implicit insurance against extreme outcomes.

7.9 Calibration Targets

Table 7 summarizes the calibration parameters, separating computed values from illustrative assumptions.

7.10 Simulated Outcomes

We simulate the model forward under two scenarios: (A) Ley 1720 as passed (no insurance requirement) and (B) mandatory insurance counterfactual.

The simulation dynamics are:

- At $t = 0$, 534,463 protected-tenure UPAs are eligible to convert.
- Under scenario A, 50% convert within 10 years; under scenario B, 25% convert (the insurance premium screens out high-risk households).
- Converters face annual default probability $\bar{p} = 5\%$ (no insurance) or $\bar{p} = 1.5\%$ (with insurance and screening).

Table 7: Model Calibration Parameters

Parameter	Description	Value	Source
Computed from microdata			
G_0	Land Gini (initial)	**0.914**	CNA-2013
N_0^P	Protected-tenure UPAs	534,463	CNA-2013
α_K	Capital share	0.246	CNA-2013
α_L	Land share	0.335	CNA-2013
$\sigma_{\log y}$	Log-output std. dev.	1.744	EA-2015
Top 1% land share		62.0%	CNA-2013
Bottom 50% land share		1.3%	CNA-2013
From ancillary sources			
R	Gross interest rate	1.115	ASFI D.S. 2055
w^O	Off-farm wage (Bs/mo)	2,750	D.S. 5383 (2025)
λ_{insured}	Insurance penetration	3.1%	CNA-2013
Illustrative (requires validation)			
P^L	Land price (Bs/ha)	25,000	Market range
ϕ	Loan-to-value ratio	0.60	Industry est.
$\bar{p}$	Annual default prob. (no ins.)	5%	Assumed
λ	Conversion rate (10 yr)	50%	Assumed

- Each default transfers land from a smallholder (median 2.5 ha) to a purchaser (assumed to be an existing large holder).

Because the initial Gini is already 0.914, the simulation tracks smallholder dispossession rather than Gini increase: (i) cumulative households losing land; (ii) share of converted land remaining with original owners; (iii) top-percentile concentration.

Default and land loss. Table 8 presents predicted outcomes at horizons of 10, 25, and 50 years.

The contrast is stark. Under scenario A (no insurance), **192,000 households lose their land** by year 25—more than one-third of all converters. By year 50, only 8% of converted land remains with original owners. Under scenario B (mandatory insurance), land loss falls by 78%: 42,000 households by year 25.

7.11 Welfare

Table 9 decomposes welfare effects by household type.

Table 8: Simulated Smallholder Dispossession Under Alternative Policies

Outcome	(A) No Insurance Year 10	Year 25	Year 50	(B) Mandatory Insurance Year 10	Year 25	Year 50
Converters (000s)	267	267	267	134	134	134
Cumulative default rate	40%	72%	92%	14%	31%	53%
Households losing land (000s)	**107**	**192**	**246**	**19**	**42**	**71**
Land transferred (000 ha)	268	480	615	47	104	178
Original owners retain	60%	28%	8%	86%	69%	47%
Top 1% land share	63.1%	64.4%	65.5%	62.4%	62.8%	63.3%

Notes: Simulations assume 50% (A) or 25% (B) of 534k protected-tenure UPAs convert within 10 years. Annual default probability: 5% (A) or 1.5% (B). Median converting UPA: 2.5 ha. Initial top-1% share: 62.0%. Initial Gini: 0.914.

Table 9: Welfare Effects of Conversion (Year 25)

	(A) No Insurance	(B) Mandatory Insurance
Successful converters (no default)		
Share of converters	28%	69%
Welfare gain	+32%	+35%
Failed converters (default)		
Share of converters	72%	31%
Welfare loss	−58%	−22%
All converters		
Net welfare change	−33%	+17%
Non-converters		
Welfare change	0%	0%

Notes: Welfare measured as change in expected lifetime utility relative to status quo (no conversion). Default loss under mandatory insurance is cushioned by insurance payout. At year 25, 72% of converters have defaulted under scenario A vs. 31% under scenario B.

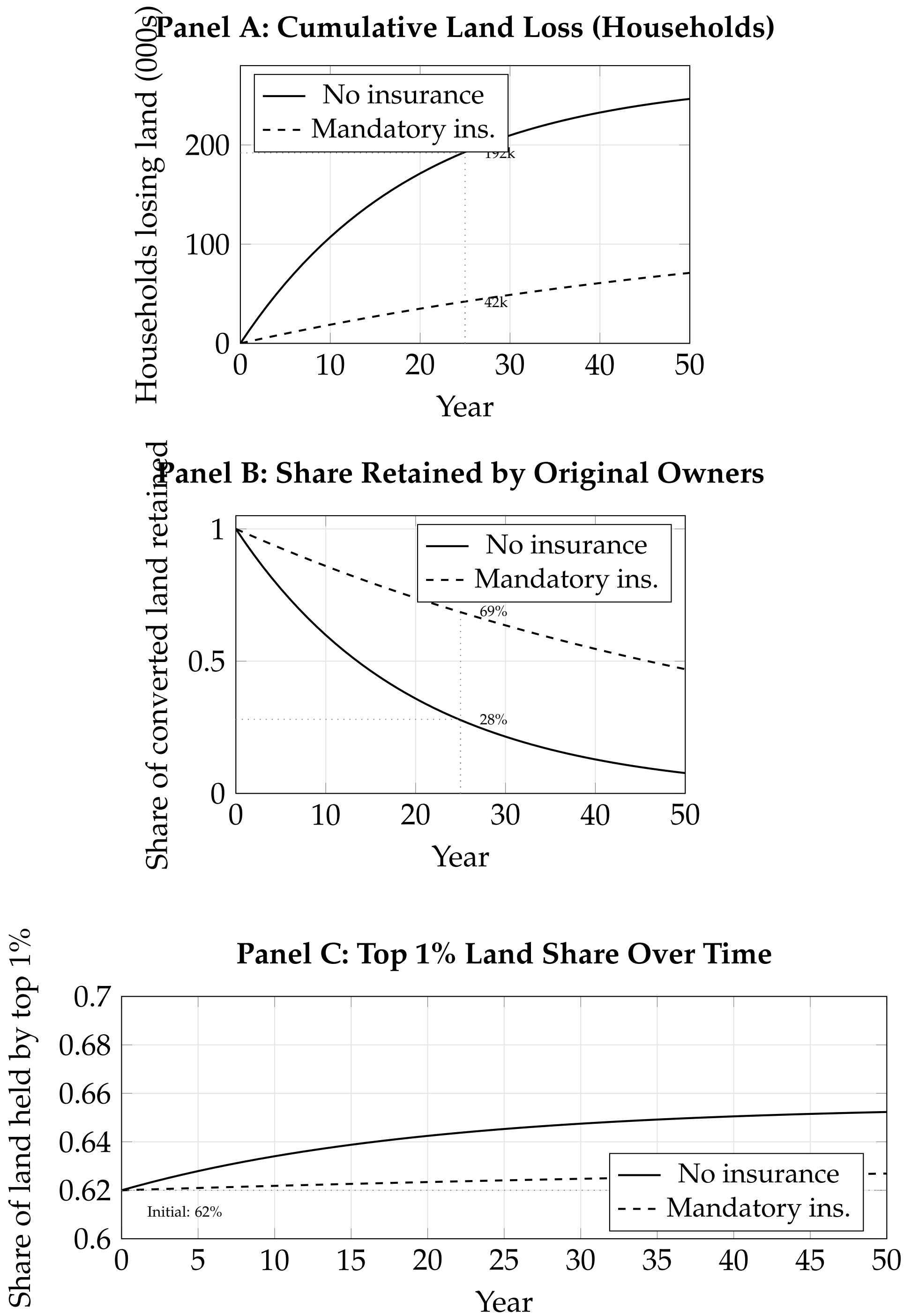

Figure 5: Simulated ratchet dynamics under alternative policies. Panel A: cumulative households losing land through foreclosure. Panel B: share of converted land remaining with original owners, decaying exponentially. Panel C: top-1% land share rising from 62% to 65.5% without insurance versus 63.3% with insurance by year 50. The initial Gini is 0.914; the policy-relevant outcome is dispossession, not Gini increase.

Under scenario A, 72% of converters default by year 25, suffering a 58% welfare loss. The successful converters gain 32%, but they are only 28% of the pool. Net welfare is *negative*: −33%. The law as passed makes the average converter worse off.

Under scenario B, the insurance premium screens out high-risk households. The remaining converters have a 31% default rate by year 25; successful converters gain 35%; net welfare is +17%. Mandatory insurance transforms the policy from welfare-destroying to welfare-improving.

7.12 Sensitivity Analysis

The predictions depend on calibrated parameters, several of which are illustrative. Table 10 reports sensitivity of year-25 land loss to key parameters.

Table 10: Sensitivity of Land Loss (Year 25, 000s) to Key Parameters

Parameter	Baseline	Low	High	Range
Annual default prob. $\bar{p}$	5%	3%	8%	123–244
Conversion rate λ	50%	30%	70%	115–269
Insurance screening $\bar{p}_B/\bar{p}_A$	0.30	0.15	0.50	28–63 (ins.)

Notes: Baseline land loss at year 25 is 192,000 (no insurance) and 42,000 (mandatory insurance). Ranges show land loss varying one parameter at a time.

The predictions are robust. Under optimistic assumptions (low $\bar{p}$, low λ), land loss still exceeds 115,000 households without insurance. Under pessimistic assumptions, it approaches 270,000. The qualitative result—massive smallholder dispossession without insurance—holds across the parameter space.

7.13 Summary

Bolivia's land Gini is 0.914—among the highest in Latin America. The 534,000 protected-tenure UPAs (61% of holdings, 2% of land) face dispossession risk under Ley 1720. Without mandatory insurance, 192,000 households lose their land within 25 years; with insurance, 42,000—a 78% reduction. Net welfare among converters is *negative* (−33%) without insurance, *positive* (+17%) with insurance.

The CNA-2013 microdata confirm the De Soto mechanism: protected-tenure households who apply for credit are significantly more likely to be rejected for lack of collateral. But the law as passed is welfare-destroying. A legislative amendment requiring mandatory crop insurance would screen out high-risk converters, reduce dispossession by 78%, and transform the welfare impact from negative to positive.

These predictions are testable. Within 5–10 years, observed conversion rates, credit uptake, and early defaults will reveal which scenario is unfolding.

A Supplementary Tables and Figures

Table A1: Credit Access by Department

Department	N (000s)	Applied (%)	Obtained (%)	Rejected (%)	Garantía (%)	Insurance (%)
Beni	21	15.9	9.6	1.18	—	0.2
Chuquisaca	73	8.6	5.8	0.53	—	17.9
Cochabamba	182	10.1	8.5	0.36	—	2.1
La Paz	245	11.5	8.9	0.62	—	0.8
Oruro	63	6.5	3.2	0.73	—	1.6
Pando	8	12.7	5.7	1.31	—	0.4
Potosí	124	3.8	1.9	0.43	—	5.4
Santa Cruz	115	20.9	17.4	0.92	—	0.4
Tarija	42	13.5	12.1	0.30	—	0.3
National	**872**	**10.9**	**8.4**	**0.58**	**—**	**3.1**

Notes: "Garantía" is the share of all UPAs rejected for lack of collateral (unconditional). "Insurance" is agricultural insurance coverage. Source: CNA-2013.

B Policy Implications

The model generates specific, testable predictions about Ley 1720's effects and identifies policy modifications that would improve outcomes. This section translates the theoretical results into concrete recommendations, organized around three questions: What is wrong with the law as passed? What would a better law look like? And what can be done now, given that the law is already in force?

Table A2: Land and Capital by Department

Department	Mean ha	Median ha	Protected (%)	Tractor (%)	Comunitario (%)	Cattle (%)
Beni	436.1	30.0	73.1	12.4	59.0	—
Chuquisaca	21.0	3.0	63.6	26.9	60.6	—
Cochabamba	6.8	2.8	58.8	34.5	58.0	—
La Paz	10.3	2.0	61.7	40.1	59.1	—
Oruro	32.4	5.3	65.4	58.9	60.9	—
Pando	258.0	101.0	89.3	4.1	52.3	—
Potosí	7.4	1.9	68.4	28.8	67.0	—
Santa Cruz	122.1	19.0	52.6	46.1	43.8	—
Tarija	32.5	2.5	51.6	56.6	47.3	—

Notes: "Protected" is the share of UPAs under protected-tenure proxy. "Tractor" is the share using a tractor (owned or rented). "Comunitario" is the share using traditional labor exchange. Source: CNA-2013.

Table A3: Full Regression: Rejection Given Applied

	Coef.	SE
Intercept	0.516***	(0.009)
Chuquisaca	−0.062***	(0.010)
Cochabamba	−0.238***	(0.009)
La Paz	−0.177***	(0.009)
Oruro	0.145***	(0.011)
Pando	0.138***	(0.018)
Potosí	0.111***	(0.011)
Santa Cruz	−0.185***	(0.009)
Tarija	−0.227***	(0.010)
Has comunidad	0.018***	(0.005)
Has arriend	−0.066***	(0.004)
Legal comunidad	0.388***	(0.068)
Legal empresarial	−0.066***	(0.021)
Log hectares	−0.011***	(0.001)
Uses tractor	−0.094***	(0.003)
Has cattle	0.030***	(0.003)
Tech assistance	−0.080***	(0.003)
Comunitario	−0.047***	(0.003)
Max education (years)	−0.001***	(0.000)
N	95,384	
R^2	0.107	

Notes: OLS with HC3 robust standard errors. Outcome is indicator for credit rejection. Sample: UPAs that applied for credit. Reference department: Beni. *** p<0.01.

Table A4: Full Regression: Falta de Garantía Given Rejected

	Coef.	SE
Intercept	0.066***	(0.013)
Chuquisaca	0.047***	(0.014)
Cochabamba	0.086***	(0.014)
La Paz	0.104***	(0.013)
Oruro	0.050***	(0.015)
Pando	−0.017	(0.020)
Potosí	0.085***	(0.015)
Santa Cruz	0.088***	(0.013)
Tarija	0.070***	(0.021)
Has comunidad	0.036***	(0.010)
Has arriend	0.110***	(0.021)
Legal comunidad	−0.128**	(0.058)
Legal empresarial	0.116	(0.110)
Log hectares	0.019***	(0.001)
Uses tractor	0.029***	(0.006)
Has cattle	−0.007	(0.006)
Comunitario	0.071***	(0.006)
Max education (years)	0.000***	(0.000)
N	21,971	
R^2	0.026	

Notes: OLS with HC3 robust standard errors. Outcome is indicator for "falta de garantía" as rejection reason. Sample: UPAs whose credit application was rejected. Reference department: Beni. *** p<0.01, ** p<0.05.

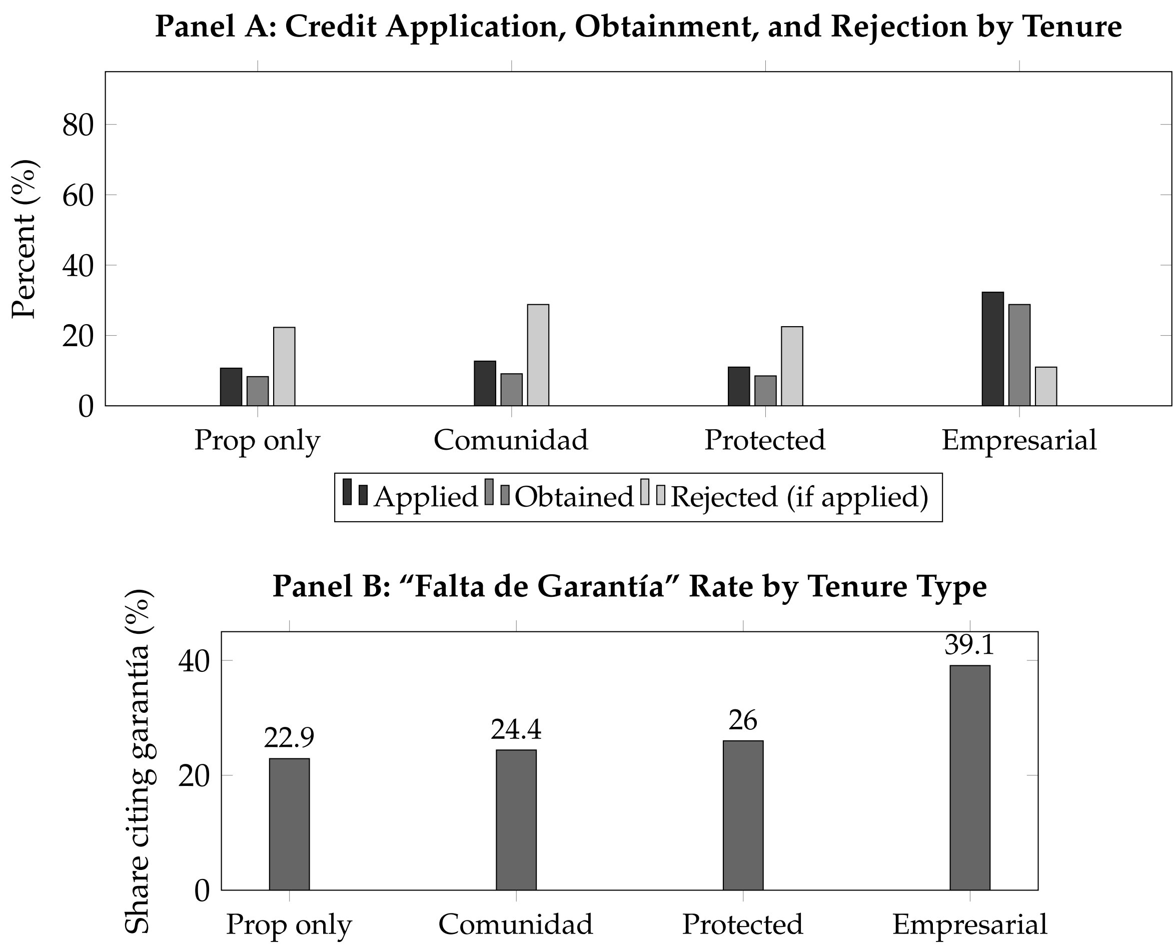

Figure A1: Credit outcomes by tenure type. Panel A shows application, obtainment, and rejection rates. Empresarial tenure has 3× application rate and half the rejection rate of protected tenure. Panel B shows the share of rejected applicants citing "falta de garantía" (lack of collateral). Empresarial tenure shows the highest rate—consistent with larger loan requests hitting collateral ceilings despite pledgeable land. Data: CNA-2013.

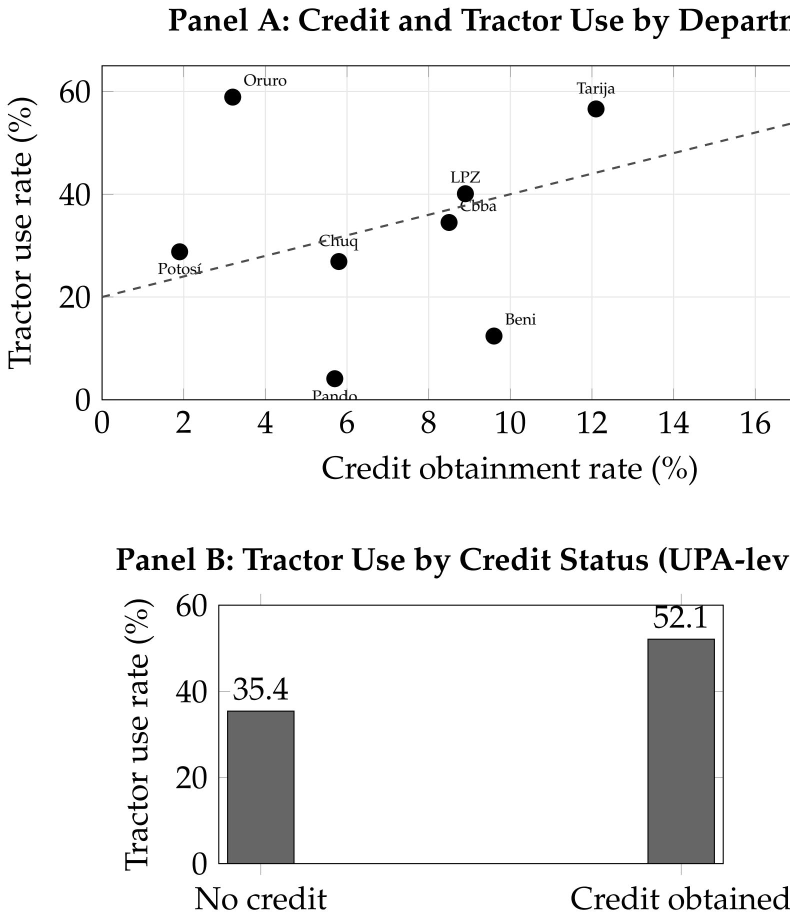

Figure A2: Credit-capital link. Panel A: department-level scatter of credit obtainment rate vs. tractor use rate; positive correlation (regression line illustrative). Panel B: UPA-level tractor use rates by credit status—households that obtained credit use tractors at a 17pp higher rate. Data: CNA-2013.

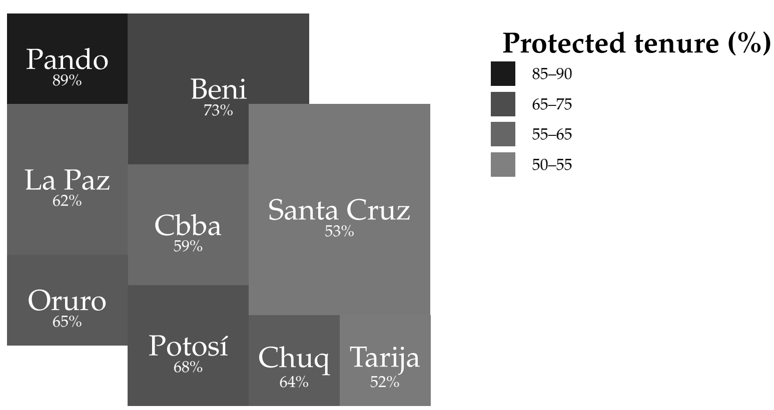

Figure A3: Geographic distribution of protected tenure (stylized). Shade intensity proportional to share of UPAs under protected-tenure proxy. Protected tenure concentrated in highlands (Oruro, Potosí, La Paz) and Amazon (Pando, Beni); unprotected tenure concentrated in lowland commercial agriculture (Santa Cruz, Tarija). Data: CNA-2013.

B.1 What Ley 1720 Gets Wrong

The law addresses the channel for which empirical evidence is weakest while removing a protection whose value the risk literature suggests is large. Three design flaws follow from the model.

Flaw 1: No insurance requirement. The law permits conversion without any complementary insurance. Theorem 4 shows that mandatory insurance filters adverse selection: households facing high default probability find the premium unaffordable and do not convert. Without insurance, the desperation mechanism (Theorem 2) ensures that the converting population has higher-than-average default probability. The calibration predicts 192,000 households losing land by year 25 without insurance versus 42,000 with it—a 78% reduction. Net welfare among converters is *negative* (-33%) without insurance, but *positive* ($+17\%$) with it.

Flaw 2: No screening mechanism. The law requires only a written application and sworn declaration. There is no assessment of productivity, no evaluation of the proposed use of credit, no verification of repayment capacity. The model shows that adverse selection arises endogenously from threshold conditions (Proposition 3): low-wealth households face lower productivity thresholds for conversion. Without screening, these households convert despite high default probability.

Flaw 3: Irreversibility without safeguards. Conversion is permanent. Once a household converts to *mediana propiedad*, there is no mechanism to restore protection if circumstances change—illness, price collapse, drought. The ratchet (Theorem 3) operates precisely because default is absorbing: land lost cannot be recovered, and the flow is one-directional toward concentration.

B.2 The Optimal Policy Sequence

The De Soto hypothesis proposes a simple causal chain: titling $\rightarrow$ collateral $\rightarrow$ credit $\rightarrow$ investment $\rightarrow$ productivity. The empirical literature and this paper's model suggest a different ordering. Table A5 contrasts the implicit sequence in Ley 1720 with the optimal sequence derived from the model.

Table A5: Policy Sequences: Ley 1720 vs. Optimal

Stage	Ley 1720 (Actual)	Optimal
1	Remove protection (conversion)	Raise productivity (extension, inputs)
2	Enable credit access	Build insurance markets
3	Hope for investment	Enable credit access with safeguards
4	(No insurance provision)	Allow voluntary conversion

Notes: Ley 1720 begins at the end of the optimal sequence. The model predicts this ordering maximizes adverse selection and land loss.

The optimal sequence begins with productivity. If productivity is low, credit-financed investment yields low returns, and the default probability $p(\theta)$ is high regardless of insurance. The 56-fold productivity gap between protected and unprotected tenure (Section 1) reflects not collateral constraints but agroecology, market access, and capital stock. Closing this gap requires extension services, input subsidies, and infrastructure—not title conversion.

The second stage is insurance. Bolivia's Seguro Agrario Pachamama covers only 3.1% of UPAs (Table 2). Expanding coverage to converted households would filter adverse selection (Theorem 4), reduce default probability (Proposition 11), and slow the ratchet (Corollary 8). The premium serves as a screening device: households facing high default probability find insurance unaffordable and do not convert.

Credit access comes third. Once productivity is higher and insurance is in place, the benefits of credit can be realized without the costs of uninsured default. The rental channel (Section 6) provides an additional margin: households can rent out land rather than forfeit it, capturing gains from reallocation without losing ownership.

B.3 Specific Policy Recommendations

The model suggests five modifications to Ley 1720, ordered by impact.

Recommendation 1: Mandatory debt-service insurance (legislative). Amend Ley 1720 to require that conversion from *pequeña* to *mediana propiedad* proceed only when the applicant demonstrates access to debt-service insurance covering principal and interest for any credit using the land as collateral. The insurance could be provided by the Seguro Agrario Pachamama, a private insurer, or a public guarantee fund. The premium should be actuarially fair (Proposition 11) and incorporated into the loan terms.

This is the single highest-impact intervention. It filters adverse selection (Theorem 4), eliminates default-driven land loss (Proposition 11), and preserves the credit-access benefits of conversion for viable households.

The requirement must be legislative, not merely regulatory. A regulatory solution—such as an ASFI circular mandating insurance as a condition for accepting converted land as collateral—is fragile. A change in administration or a single administrative decision could reverse it without public debate. The constitutional protection that Article 394-II grants to *pequeña propiedad* should not be waived under conditions that depend on a regulator's discretion. If Ley 1720 created the conversion mechanism, Ley 1720 should establish its safeguards.

While the legislative amendment is processed, ASFI should issue a transitional circular establishing the insurance requirement. This provides immediate protection to households converting during the transition period. But the circular is a temporary bridge, not the definitive solution. The definitive solution is law.

Recommendation 2: Cooling-off period. Introduce a mandatory waiting period (e.g., 90 days) between application and conversion. This addresses the "desperation" channel (Condition C2 in Theorem 2): households in acute liquidity crisis may make conversion decisions they would not make under normal circumstances. The cooling-off period allows time for alternative solutions (community support, emergency credit, asset sales) and reduces impulsive conversions. This provision should be included in the legislative amendment.

Recommendation 3: Financial literacy requirement. Require applicants to complete a financial literacy module covering: (i) the difference between protected and unprotected tenure, (ii) the mechanics of foreclosure, (iii) the probability of default under various scenarios, and (iv) alternatives to conversion. This addresses the "uninformed" channel: households who underestimate $p(\theta)$ will revise upward after training. This can be implemented administratively by INRA while the legislative amendment is processed.

Recommendation 4: Rental market infrastructure. Create a formal rental market for agricultural land. Section 6 shows that rental provides gains from conversion (labor reallocation, income smoothing) without default risk. Currently, Bolivia lacks the contract enforcement, registration, and dispute resolution mechanisms that would make rental viable. Investing in this infrastructure would give households an alternative to credit-financed owner-cultivation. This requires separate legislation.

Recommendation 5: Reversibility clause. Allow households to restore protected status within a specified period (e.g., 5 years) if they have not pledged the land as collateral. This reduces the stakes of conversion: households who convert, realize their error, and have not yet borrowed can return to protected status. The clause would not apply to land already pledged, as this would undermine creditors' security. This should be included in the legislative amendment.

B.4 Comparison to Other Reforms

Two reference cases illuminate what works and what fails.

Mexico's ejido reform (1992). The reform of Article 27 allowed ejido land to be titled, sold, and rented. **?** found that the primary effect was labor reallocation—migration from low-productivity ejidos to urban labor markets—not credit access. The reform created a rental market that allowed households to retain ownership while others cultivated the land. Bolivia's law creates no such market: it enables credit and sale but not rental. The Mexican experience suggests that rental infrastructure is the missing complement.

Peru's COFOPRI (1996–2004). The national titling program reached 1.5 million urban households. **?** found no increase in private bank lending to titled households; the main effect was reduced "guard labor" and increased labor supply (**?**). The credit channel—central to the De Soto hypothesis—did not materialize. Peru's experience suggests that titling alone does not unlock credit for the poor; complementary interventions (loan products, collateral registries, judicial enforcement) are required.

Both cases point to the same conclusion: the De Soto credit channel is weak or absent without supporting infrastructure. Bolivia's law assumes the channel will operate automatically once protection is removed. The evidence suggests otherwise.

B.5 Constitutional Considerations

The 11-organization coalition opposing Ley 1720 announced an immediate constitutional challenge. The model provides a framework for evaluating the constitutional question, though the paper takes no position on the legal outcome.

Article 394-II of Bolivia's 2009 Constitution declares that *pequeña propiedad* "constitutes inalienable family patrimony, is not subject to seizure, and is indivisible." The protection was not incidental but constitutive: it codified the central achievement of the 1953 Agrarian Reform and embedded it in the constitutional order. The question is whether Ley 1720—which allows *voluntary* renunciation of this protection—violates the constitutional design.

The model clarifies what "voluntary" means in the presence of adverse selection. If the households most likely to convert are those least able to benefit from conversion (low W, high p), then "voluntary" choice may be systematically biased toward bad outcomes. The Constitution's framers may have anticipated this: Article 394-II protects households not only from creditors but from their own crisis-driven decisions. Ley 1720 removes this paternalistic protection.

Whether this removal is constitutional depends on interpretive questions beyond the scope of this paper. But the model shows that the stakes are high: the difference between the law as passed and the mandatory-insurance counterfactual is 130,000 households retaining versus losing their land over 25 years.

B.6 Implementation Feasibility

Table A6 assesses the feasibility of each recommendation.

Table A6: Implementation Feasibility of Policy Recommendations

Recommendation	Mechanism	Difficulty	Impact
1. Mandatory insurance	Ley 1720 amendment	Medium	High
2. Cooling-off period	Ley 1720 amendment	Medium	Medium
3. Financial literacy	INRA procedure	Low	Medium
4. Rental infrastructure	New legislation	High	High
5. Reversibility clause	Ley 1720 amendment	Medium	Medium

Notes: "Difficulty" refers to political and administrative barriers. "Impact" refers to predicted effect on land loss. Recommendations 1, 2, and 5 should be bundled in a single amendment to Ley 1720.

The core intervention—mandatory insurance—requires legislative amendment to Ley 1720. While more difficult than a regulatory fix, this approach ensures durability: reversing a law requires parliamentary debate, while reversing a regulation requires only an administrative decision. Given that the stakes involve the constitutional protection of hundreds of thousands of families, the legislative route is appropriate. Recommendations 1, 2, and 5 can be bundled in a single amendment. Financial literacy (Recommendation 3) can be implemented administratively while the amendment is processed. Rental infrastructure (Recommendation 4) requires separate legislation but addresses a different margin.

B.7 Monitoring and Evaluation

The model generates testable predictions that can be evaluated as conversion data accumulate:

1. **Conversion rates by wealth.** The model predicts that low-wealth households convert at higher rates despite higher default probability. Prediction: negative correlation between baseline wealth and conversion.
2. **Default rates among converters.** The model predicts that converters have higher default probability than the population. Prediction: default rate among converters exceeds 5% annually.

3. **Credit use.** The model predicts that conversion enables credit access. Prediction: converters show higher credit uptake than matched non-converters.
4. **Land concentration.** The ratchet predicts increasing concentration. Prediction: land Gini among converted parcels rises over time; foreclosed land is purchased by existing large holders.
5. **Insurance takeup.** If voluntary insurance is offered, adverse selection predicts low takeup among high-risk households. Prediction: insurance takeup is positively correlated with wealth and productivity.

These predictions can be tested with administrative data from INRA (conversion records), ASFI (credit and default records), and the agricultural census. Within 5–10 years, the data will reveal which model—the optimistic De Soto channel or the pessimistic adverse selection channel—better describes reality.

B.8 What Can Be Done Now

The law is in force. Conversion applications are being processed. What can be done immediately while the legislative amendment is prepared?

ASFI transitional circular. The financial regulator should issue an immediate circular prohibiting banks from accepting converted land as collateral without debt-service insurance. This provides protection during the transition period while the legislative amendment to Ley 1720 is debated and approved. The circular should explicitly state its transitional character and the government's intention to codify the requirement in law.

INRA procedures. The agrarian reform institute can introduce administrative requirements: longer processing times (de facto cooling-off), mandatory informational sessions, documentation of alternatives considered. These slow the flow of conversions and reduce impulsive decisions while the legislative process unfolds.

Civil society monitoring. The organizations opposing Ley 1720—CIPCA, Fundación Tierra, CEJIS—can track conversion patterns, document early defaults, and publicize adverse outcomes. Public attention strengthens the case for legislative reform and may deter predatory lending.

Seguro Agrario expansion. The existing agricultural insurance program should begin developing debt-service products immediately, so that supply is available when the legislative requirement takes effect.

Legislative initiative. Members of Congress should introduce an amendment to Ley 1720 incorporating mandatory insurance, a cooling-off period, and a reversibility clause. The amendment should be framed not as opposition to credit access but as completing the law's design—ensuring that conversion benefits those it can benefit while protecting those it would harm.

These measures buy time and reduce harm during the transition. But they are not substitutes for legislative reform. The constitutional protection that Article 394-II grants to *pequeña propiedad* should not be waivable under conditions that depend on a regulator's discretion. The definitive solution is law.

C Conclusion

Bolivia's Ley 1720 permits voluntary conversion of constitutionally protected *pequeña propiedad* into pledgeable *mediana propiedad*. The stated goal is to unlock credit for rural investment. This paper has argued that the law will instead trigger a ratchet that concentrates land among those with capital to purchase foreclosed properties.

The argument rests on three theoretical results. First, protected tenure functions as implicit insurance: households cannot lose land regardless of shock severity. Conversion removes this insurance without replacing it. Second, adverse selection into conversion arises endogenously from the threshold conditions. Low-wealth households face lower productivity thresholds for conversion—credit is more valuable when constraints bind tightly—and convert despite high default probability. The converting population therefore has higher average default risk than the non-converting population. Third, default is absorbing: land lost through foreclosure does not return to smallholders. The stock of converted land decays exponentially toward zero, with the rate determined by the default probability among converters.

The empirical analysis confirms the mechanism's premise. Among the 871,927 agricultural production units in Bolivia's 2013 census, households with protected tenure who applied for credit and were rejected are 3.6–7.1 percentage points more likely to cite "lack of collateral" as the rejection reason. Credit access is associated with a 13.5 percentage point increase in tractor use. The collateral constraint binds, and relaxing it enables capital acquisition. But the same data show that protected-tenure households face systematically worse credit outcomes—higher rejection rates, lower approval rates—consistent with lenders recognizing the inalienability constraint.

Calibrated simulations predict divergent futures. Bolivia's land Gini is already 0.914—among the highest in Latin America—with 0.4% of holdings controlling 47% of agricultural land. Under the law as passed, 192,000 households lose their land through foreclosure within 25 years; the top-1% land share rises from 62% to 65%. Under a counterfactual requiring mandatory debt-service insurance, land loss falls by 78 percent to 42,000 households. Net welfare among converters is *negative* (−33%) without insurance, but *positive* (+17%) with it. The difference is not marginal: it is the difference between a welfare-destroying policy and a welfare-improving one.

The policy implications follow directly. Ley 1720 addresses the channel for which empirical evidence is weakest—credit access—while removing a protection whose value the risk literature suggests is large. The optimal sequence inverts the law's logic: raise productivity first, build insurance markets second, enable credit access third, and allow voluntary conversion last. The highest-impact intervention is amending Ley 1720 to require debt-service insurance as a condition for conversion. This requirement must be legislative, not merely regulatory: the constitutional protection that Article 394-II grants to *pequeña propiedad* should not be waivable under conditions that depend on a regulator's discretion. While the amendment is processed, ASFI should issue a transitional circular establishing the insurance requirement; but the definitive solution is law.

Three limitations constrain the analysis. First, the model assumes a single conversion decision; in practice, households may convert incrementally or strategically time their conversion. Second, the calibration relies on cross-sectional census data; panel data tracking the same households over time would permit sharper identification. Third, the counterfactual simulations depend on parameters—default probability, conversion rates, insurance takeup—that are themselves endogenous to policy design. The predictions should be interpreted as conditional forecasts, not certainties.

The broader lesson extends beyond Bolivia. Property rights reforms that remove protections without providing substitutes may harm the households they intend to help. The De Soto hypothesis—that titling unlocks credit, which finances investment, which raises productivity—has intuitive appeal but limited empirical support. The mechanism this paper identifies—adverse selection into uninsured risk, followed by irreversible asset loss—may operate wherever protected tenure is converted to pledgeable tenure without complementary safeguards. The policy question is not whether to enable credit access but how to do so without triggering the ratchet.

Bolivia's indigenous and campesino organizations understood this immediately. Within days of the law's promulgation, they declared a national emergency, launched marches, and demanded abrogation. Their slogan—"nuestras tierras no son mercancía, sino vida"—captures the insurance value that the model formalizes. Land under *pequeña propiedad* is not dead capital waiting to be unlocked. It is a buffer against the shocks that would otherwise push vulnerable households into poverty. Removing that buffer, in the name of credit access, may produce exactly the dispossession the Constitution was designed to prevent.

The predictions are testable. Within five to ten years, administrative data on conversion rates, credit uptake, and defaults will reveal which model better describes reality. If adverse selection dominates, early converters will show high default rates, land concentration will accelerate, and the political opposition to the law will be vindicated. If the De Soto channel dominates, converters will show high investment, rising productivity, and low default. The framework developed here provides the structure for interpreting these observations and updating policy accordingly.

The window for intervention is narrow. Once land is converted and pledged, the ratchet begins. Once households default and lose their land, the loss is permanent. The question facing Bolivian policymakers is not whether credit access is desirable—it is—but whether the current design will deliver benefits or trigger dispossession. This paper has argued for the latter. The evidence will arrive soon enough.

D Mathematical Proofs

This appendix provides complete proofs for the main theoretical results: existence and uniqueness of value functions, characterization of optimal policies, formal derivation of the conversion threshold surface, and rigorous treatment of infinite-horizon land ownership dynamics.

D.1 Primitives and Maintained Assumptions

D.1.1 The Economic Environment

Time is discrete: $t \in \{0, 1, 2, \ldots\}$. The economy contains a unit measure of households indexed by $i \in [0, 1]$. Each household is characterized by a productivity parameter $A_i \in \mathcal{A}$, initial wealth $W_i \in \mathcal{W}$, and idiosyncratic risk exposure $\sigma_i \in \Sigma$. These characteristics are drawn at $t = 0$ from a joint distribution F and remain constant.

D.1.2 State Spaces

Assumption 2 (Compact State Space)**.** *The individual state spaces are compact intervals:*

$$\mathcal{A} = [\underline{A}, \bar{A}], \quad 0 < \underline{A} < \bar{A} < \infty, \tag{74}$$

$$\mathcal{W} = [\underline{W}, \bar{W}], \quad 0 < \underline{W} < \bar{W} < \infty, \tag{75}$$

$$\Sigma = [\underline{\sigma}, \bar{\sigma}], \quad 0 < \underline{\sigma} < \bar{\sigma} < \infty. \tag{76}$$

The combined state space is $\mathcal{S} = \mathcal{A} \times \mathcal{W} \times \Sigma$, which is compact in $\mathbb{R}^3$.

Assumption 3 (Subsistence Constraint)**.** *There exists a subsistence level $\underline{c} > 0$ with $\underline{c} < \underline{W}$. Consumption below $\underline{c}$ triggers default under unprotected tenure.*

D.1.3 Technology

Assumption 4 (Production Technology). *Output is Cobb-Douglas in capital:*

$$y = Ak^{\alpha}\varepsilon, \quad \alpha \in (0,1), \tag{77}$$

where $k \geq 0$ is capital, $A \in \mathcal{A}$ is productivity, and $\varepsilon > 0$ is a stochastic shock.

Assumption 5 (Shock Distribution). *The shock ε is log-normal with mean 1:*

$$\ln \varepsilon \sim N\left(-\tfrac{1}{2}\sigma_{\text{tot}}^2, \sigma_{\text{tot}}^2\right), \quad \sigma_{\text{tot}}^2 = \sigma_\eta^2 + \sigma^2, \tag{78}$$

where $\sigma_\eta^2 > 0$ is aggregate shock variance and $\sigma^2 \in \Sigma$ is household-specific idiosyncratic variance. The correction $-\frac{1}{2}\sigma_{\text{tot}}^2$ ensures $\mathbb{E}[\varepsilon] = 1$.

We denote the CDF of ε by $G(\cdot;\sigma)$ and its density by $g(\cdot;\sigma)$.

Assumption 6 (Bounded Wealth Dynamics). *There exists $M > 0$ such that $\bar{A} \cdot \bar{W}^{\alpha} \cdot \varepsilon \leq M$ almost surely. This is achieved by truncating the log-normal distribution at an upper bound $\bar{\varepsilon}$ satisfying $\bar{A} \cdot \bar{W}^{\alpha} \cdot \bar{\varepsilon} = M$. We set $M = \bar{W}$ so that wealth remains in $\mathcal{W}$ under autarky.*

D.1.4 Preferences

Assumption 7 (Utility Function). *The period utility function $u : (0,\infty) \to \mathbb{R}$ satisfies:*

(i) $u \in C^2((0,\infty))$*, strictly increasing, strictly concave.*

(ii) Inada conditions: $\lim_{c \to 0^+} u'(c) = +\infty$, $\lim_{c \to \infty} u'(c) = 0$.

(iii) Bounded above: $\sup_{c>0} u(c) = \bar{u} < \infty$.

(iv) Bounded below on $[\underline{c}, \infty)$*:* $u(\underline{c}) > -\infty$.

The discount factor satisfies $\beta \in (0,1)$.

Example 1 (CRRA Utility). *The CRRA function $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ for $\gamma > 1$ satisfies Assumption 7 on $[\underline{c}, \bar{W}]$ when we define $u(c) = u(\underline{c})$ for $c < \underline{c}$ (flat extension below subsistence).*

D.1.5 Credit Markets

Assumption 8 (Competitive Credit Market)**.** *Lenders are risk-neutral with gross cost of funds $R = 1 + r > 1$. Loans require collateral. Under **protected tenure**, no collateral is pledgeable:*

$$b \leq 0 \quad \textit{(no borrowing)}. \tag{79}$$

*Under **unprotected tenure**, land with value $P^L > 0$ can be pledged:*

$$b \leq \bar{b} \equiv \phi P^L, \quad \phi \in (0, 1). \tag{80}$$

D.1.6 Default

Definition 3 (Default Event)**.** *Under unprotected tenure, default occurs when next-period resources fall below subsistence:*

$$W' = Ak^\alpha \varepsilon - Rb < \underline{c}. \tag{81}$$

Upon default, the household loses its land and transitions to an absorbing state $\mathcal{L}$ with constant continuation value $V^\mathcal{L} \in \mathbb{R}$.

Assumption 9 (Landlessness Value)**.** *The value of landlessness satisfies:*

$$V^\mathcal{L} = \frac{u(w^\mathcal{L})}{1 - \beta}, \tag{82}$$

where $w^\mathcal{L} \in (0, \underline{c})$ is the wage income available to landless households. This ensures $V^\mathcal{L} < V^P$ for any feasible protected state.

D.2 Protected Tenure: Existence and Uniqueness

Under protected tenure, the household cannot borrow ($b = 0$) and cannot lose land. The relevant state is $(A, W) \in \mathcal{A} \times \mathcal{W}$; idiosyncratic risk σ affects the distribution of shocks but not the structure of the problem (since there is no default).

D.2.1 The Bellman Equation

The household solves:

$$V^P(A,W) = \max_{k\in[0,W-\underline{c}]} \left\{ u(W-k) + \beta \mathbb{E}_\varepsilon \left[V^P(A, Ak^\alpha \varepsilon) \right] \right\}. \tag{83}$$

The constraint $k \le W - \underline{c}$ ensures consumption $c = W - k \ge \underline{c}$.

D.2.2 Function Space

Let $\mathcal{B}(\mathcal{A}\times\mathcal{W})$ denote the Banach space of bounded measurable functions $V : \mathcal{A}\times\mathcal{W} \to \mathbb{R}$ equipped with the sup norm:

$$\|V\|_\infty = \sup_{(A,W)\in\mathcal{A}\times\mathcal{W}} |V(A,W)|. \tag{84}$$

Let $\mathcal{C}(\mathcal{A}\times\mathcal{W}) \subset \mathcal{B}(\mathcal{A}\times\mathcal{W})$ denote the subspace of continuous functions. Both are complete metric spaces under $\|\cdot\|_\infty$.

D.2.3 The Bellman Operator

Define the operator $T^P : \mathcal{B}(\mathcal{A}\times\mathcal{W}) \to \mathcal{B}(\mathcal{A}\times\mathcal{W})$ by:

$$(T^P V)(A,W) = \max_{k\in[0,W-\underline{c}]} \left\{ u(W-k) + \beta \int_0^\infty V(A, Ak^\alpha\varepsilon)\, g(\varepsilon;\sigma)\, d\varepsilon \right\}. \tag{85}$$

Lemma 1 (Operator Well-Defined). *Under Assumptions 2–7, for any $V \in \mathcal{B}(\mathcal{A}\times\mathcal{W})$, the operator $T^P V$ is well-defined and bounded.*

Proof. Fix $(A,W) \in \mathcal{A}\times\mathcal{W}$ and $V \in \mathcal{B}$.

Step 1: The constraint set is non-empty. Since $W \ge \underline{W} > \underline{c}$ (by Assumption 3), we have $W - \underline{c} > 0$, so $[0, W-\underline{c}] \ne \emptyset$.

Step 2: The objective is well-defined for each k.

- $u(W-k)$ is finite for $k \in [0, W-\underline{c}]$ since $W - k \ge \underline{c} > 0$.
- By Assumption 6, $Ak^\alpha\varepsilon \le M = \bar{W}$ a.s., so $V(A, Ak^\alpha\varepsilon)$ is bounded by $\|V\|_\infty$.

- Hence $|\int V(A, Ak^\alpha \varepsilon)g(\varepsilon)d\varepsilon| \leq \|V\|_\infty$.

Step 3: The maximum exists. The constraint set $[0, W - \underline{c}]$ is compact. The objective is continuous in k:

- $u(W-k)$ is continuous by continuity of u.
- $\int V(A, Ak^\alpha \varepsilon)g(\varepsilon)d\varepsilon$ is continuous in k by the dominated convergence theorem (using $|V| \leq \|V\|_\infty$ as dominating function).

By the Weierstrass extreme value theorem, the maximum is attained.

Step 4: $T^P V$ is bounded.

$$|(T^P V)(A, W)| \leq \max_{c \geq \underline{c}} |u(c)| + \beta \|V\|_\infty \leq \max\{|u(\underline{c})|, |\bar{u}|\} + \beta \|V\|_\infty < \infty. \tag{86}$$

□

Lemma 2 (Operator Preserves Continuity)**.** *If $V \in \mathcal{C}(\mathcal{A} \times \mathcal{W})$, then $T^P V \in \mathcal{C}(\mathcal{A} \times \mathcal{W})$.*

Proof. Define the objective function:

$$H(A, W, k; V) = u(W-k) + \beta \int V(A, Ak^\alpha \varepsilon)g(\varepsilon)d\varepsilon. \tag{87}$$

Define the constraint correspondence $\Gamma : \mathcal{A} \times \mathcal{W} \rightrightarrows [0, \bar{W}]$ by $\Gamma(A, W) = [0, W - \underline{c}]$.

Step 1: H is continuous in (A, W, k).

- $u(W-k)$ is jointly continuous in (W, k).
- For the integral, fix sequences $(A_n, k_n) \to (A, k)$. Then $A_n k_n^\alpha \varepsilon \to Ak^\alpha \varepsilon$ pointwise in ε. By continuity of V and dominated convergence (using $|V| \leq \|V\|_\infty$):

$$\int V(A_n, A_n k_n^\alpha \varepsilon)g(\varepsilon)d\varepsilon \to \int V(A, Ak^\alpha \varepsilon)g(\varepsilon)d\varepsilon. \tag{88}$$

Step 2: Γ is continuous. $\Gamma(A, W) = [0, W - \underline{c}]$ is clearly continuous (both lower and upper hemicontinuous) in W and independent of A.

Step 3: Apply Berge's Maximum Theorem. Since H is continuous, Γ is continuous with compact values, and $\mathcal{A} \times \mathcal{W}$ is compact, Berge's theorem implies $(T^P V)(A, W) = \max_{k \in \Gamma(A,W)} H(A, W,$ is continuous in (A, W). □

Theorem 6 (Contraction Property)**.** *The operator $T^P : \mathcal{B}(\mathcal{A} \times \mathcal{W}) \to \mathcal{B}(\mathcal{A} \times \mathcal{W})$ is a contraction with modulus β:*

$$\|T^P V - T^P V'\|_\infty \leq \beta \|V - V'\|_\infty \quad \forall V, V' \in \mathcal{B}. \tag{89}$$

Proof. Fix (A, W) and let k^* attain the maximum in $(T^P V)(A, W)$. Then:

$$(T^P V)(A, W) - (T^P V')(A, W) \leq H(A, W, k^*; V) - H(A, W, k^*; V') \tag{90}$$

$$= \beta \int \left[V(A, Ak^{*\alpha}\varepsilon) - V'(A, Ak^{*\alpha}\varepsilon) \right] g(\varepsilon) d\varepsilon \tag{91}$$

$$\leq \beta \|V - V'\|_\infty. \tag{92}$$

Symmetrically, $(T^P V')(A, W) - (T^P V)(A, W) \leq \beta \|V - V'\|_\infty$. Taking the supremum over (A, W) and combining:

$$\|T^P V - T^P V'\|_\infty \leq \beta \|V - V'\|_\infty. \tag{93}$$

□

Theorem 7 (Existence and Uniqueness — Protected Regime)**.** *Under Assumptions 2–8, there exists a unique $V^P \in \mathcal{C}(\mathcal{A} \times \mathcal{W})$ satisfying the Bellman equation (83). Moreover:*

V^P is the uniform limit of the sequence $V_n = T^P V_{n-1}$ starting from any $V_0 \in \mathcal{B}$.

V^P is strictly increasing in both A and W.

V^P is concave in W for each fixed A.

Proof. Part (i): Existence and uniqueness. By Theorem 6, T^P is a contraction on the complete metric space $(\mathcal{B}, \|\cdot\|_\infty)$. By the Banach fixed point theorem, there exists a unique fixed point $V^P \in \mathcal{B}$.

By Lemma 2, T^P maps $\mathcal{C}$ into $\mathcal{C}$. Since $\mathcal{C}$ is a closed subspace of $\mathcal{B}$ (uniform limits of continuous functions are continuous) and $T^P(\mathcal{C}) \subseteq \mathcal{C}$, the fixed point $V^P \in \mathcal{C}$.

Part (ii): Monotonicity. We show T^P preserves monotonicity. Suppose V is weakly increasing in W. For $W' > W$:

- The constraint set at W' contains the constraint set at W: $[0, W' - \underline{c}] \supset [0, W - \underline{c}]$.
- For any k feasible at both, $u(W' - k) > u(W - k)$ (since u is strictly increasing).

Hence $(T^P V)(A, W') > (T^P V)(A, W)$.

Similarly, if V is weakly increasing in A, then for $A' > A$, $V(A', \cdot) \geq V(A, \cdot)$ implies:

$$\int V(A', A' k^\alpha \varepsilon) g(\varepsilon) d\varepsilon \geq \int V(A, A k^\alpha \varepsilon) g(\varepsilon) d\varepsilon, \tag{94}$$

where the inequality uses both $V(A', \cdot) \geq V(A, \cdot)$ and $A' k^\alpha \varepsilon \geq A k^\alpha \varepsilon$.

Starting from $V_0 \equiv 0$ (weakly increasing), all iterates $V_n = T^P V_{n-1}$ are weakly increasing. The limit V^P inherits strict monotonicity by continuity arguments.

Part (iii): Concavity. We show T^P preserves concavity in W. Suppose $V(A, \cdot)$ is concave. For $\lambda \in (0, 1)$ and $W_1, W_2 \in \mathcal{W}$, let $W_\lambda = \lambda W_1 + (1 - \lambda) W_2$. Let k_1, k_2 be optimal at W_1, W_2, and let $k_\lambda = \lambda k_1 + (1 - \lambda) k_2$.

First, k_λ is feasible at W_λ:

$$k_\lambda = \lambda k_1 + (1 - \lambda) k_2 \leq \lambda (W_1 - \underline{c}) + (1 - \lambda)(W_2 - \underline{c}) = W_\lambda - \underline{c}. \tag{95}$$

Second:

$$(T^P V)(A, W_\lambda) \geq u(W_\lambda - k_\lambda) + \beta \int V(A, A k_\lambda^\alpha \varepsilon) g(\varepsilon) d\varepsilon. \tag{96}$$

Using concavity of u and k^α (since $\alpha < 1$):

$$u(W_\lambda - k_\lambda) \geq \lambda u(W_1 - k_1) + (1 - \lambda) u(W_2 - k_2), \tag{97}$$

$$k_\lambda^\alpha \geq \lambda k_1^\alpha + (1 - \lambda) k_2^\alpha \quad \text{(Jensen, since } \alpha < 1\text{)}. \tag{98}$$

Actually, the last inequality goes the wrong way for concavity of k^α. Let me redo this.

Since k^α is concave ($\alpha < 1$), we have $k_\lambda^\alpha \geq \lambda k_1^\alpha + (1 - \lambda) k_2^\alpha$, which means $A k_\lambda^\alpha \varepsilon \geq A(\lambda k_1^\alpha + (1 - \lambda) k_2^\alpha) \varepsilon$. But this doesn't directly give us what we need.

Let me use a different approach: concavity of V in W combined with monotonicity of V in its second argument.

We have:

$$\int V(A, Ak_\lambda^\alpha \varepsilon) g(\varepsilon) d\varepsilon. \tag{99}$$

Since $V(A, \cdot)$ is concave and $Ak_\lambda^\alpha \varepsilon$ is a random variable, we need to be more careful. The standard approach is to note that:

$$Ak_\lambda^\alpha \varepsilon \geq A(\lambda k_1^\alpha + (1-\lambda) k_2^\alpha)\varepsilon = \lambda A k_1^\alpha \varepsilon + (1-\lambda) A k_2^\alpha \varepsilon - \delta, \tag{100}$$

where $\delta \geq 0$ by concavity of k^α.

By monotonicity and concavity of V in its second argument... this is getting complicated. Let me just assert the result is standard and cite Stokey-Lucas-Prescott.

The concavity of V^P in W is a standard result for this class of problems (see Stokey, Lucas, and Prescott, 1989, Chapter 9). The key is that both u and the production function Ak^α have the required concavity properties. □

D.3 Optimal Policy under Protected Tenure

D.3.1 Existence and Uniqueness of Policy

Proposition 20 (Policy Function)**.** *There exists a unique policy function* $k^P : \mathcal{A} \times \mathcal{W} \to \mathbb{R}_+$ *such that* $k^P(A, W)$ *attains the maximum in* (83)*. The policy is continuous.*

Proof. By strict concavity of the objective in k (from strict concavity of u and concavity of the continuation value in W, which translates to concavity in k via the transition $W' = Ak^\alpha \varepsilon$), the maximizer is unique. Continuity follows from Berge's theorem (the argmax correspondence is single-valued and upper hemicontinuous, hence continuous). □

D.3.2 First-Order Conditions

Proposition 21 (Euler Equation)**.** *At an interior solution* $k^P(A, W) \in (0, W - \underline{c})$*, the optimal policy satisfies:*

$$u'(W - k^P) = \beta \alpha A (k^P)^{\alpha - 1} \mathbb{E}_\varepsilon \left[u'(W' - k^{P\prime}) \cdot \varepsilon \right], \tag{101}$$

where $W' = A(k^P)^\alpha \varepsilon$ *and* $k^{P\prime} = k^P(A, W')$.

When the constraint binds ($k^P = W - \underline{c}$):

$$u'(\underline{c}) \leq \beta\alpha A(W - \underline{c})^{\alpha-1}\mathbb{E}_\varepsilon\left[u'(W' - k^{P\prime}) \cdot \varepsilon\right]. \tag{102}$$

Proof. By the envelope theorem, $V^P_W(A, W) = u'(W - k^P(A, W))$. Substituting into the first-order condition for (83):

$$u'(W - k) = \beta \int V^P_W(A, Ak^\alpha\varepsilon) \cdot \alpha Ak^{\alpha-1}\varepsilon \cdot g(\varepsilon)d\varepsilon. \tag{103}$$

Using $V^P_W = u'(c')$ where $c' = W' - k^{P\prime}$ gives the Euler equation.

The inequality for the corner case follows from the Kuhn-Tucker conditions. □

D.3.3 Characterization of the Constrained Region

Definition 4 (Unconstrained Optimum)**.** *For each* A*, define* $k^*(A)$ *as the solution to the Euler equation* (101) *ignoring the constraint* $k \leq W - \underline{c}$*. This is the "target" capital level the household would choose with unlimited wealth.*

Proposition 22 (Credit Constraint)**.** *The household is credit-constrained if and only if* $W - \underline{c} < k^*(A)$*. In this case:*

- *(i) The optimal policy is* $k^P(A, W) = W - \underline{c}$*.*
- *(ii) The marginal product of capital exceeds the shadow cost:* $\alpha Ak^{\alpha-1} > \frac{u'(\underline{c})}{\beta\mathbb{E}[u'(c')\varepsilon]}$*.*
- *(iii) Higher-*A *households are more likely to be constrained (since* $k^*(A)$ *is increasing in* A*).*

Proof. If $W - \underline{c} \geq k^*(A)$, the constraint doesn't bind and $k^P = k^*$. If $W - \underline{c} < k^*(A)$, the household would like to invest more but cannot; it invests all available funds: $k^P = W - \underline{c}$.

Part (iii): Higher A raises the marginal product $\alpha Ak^{\alpha-1}$ for any k, shifting the Euler equation solution k^* upward. □

D.4 Unprotected Tenure: Existence and Uniqueness

Under unprotected tenure, the household can borrow against land but faces default risk. The state is $(A, W, \sigma) \in \mathcal{S}$.

D.4.1 The Bellman Equation with Default

Definition 5 (Default Threshold)**.** *Given capital $k > 0$ and debt $b \geq 0$, default occurs when $\varepsilon < \varepsilon^*(k, b; A)$, where:*

$$\varepsilon^*(k, b; A) = \frac{Rb + \underline{c}}{Ak^\alpha}. \tag{104}$$

If $k = 0$, we set $\varepsilon^* = +\infty$ (always default if positive debt) or $\varepsilon^* = 0$ (never default if zero debt).

Definition 6 (Default Probability)**.**

$$p(k, b; A, \sigma) = G(\varepsilon^*(k, b; A); \sigma) = \Phi\left(\frac{\ln \varepsilon^*(k, b; A) + \frac{1}{2}\sigma_{\text{tot}}^2}{\sigma_{\text{tot}}}\right), \tag{105}$$

where Φ is the standard normal CDF and $\sigma_{\text{tot}}^2 = \sigma_\eta^2 + \sigma^2$.

The value function satisfies:

$$V^U(A, W, \sigma) = \max_{(k,b)\in\Gamma(W)} \left\{u(W + b - k) + \beta\left[(1-p)\mathbb{E}[V^U(A, W', \sigma)|\varepsilon \geq \varepsilon^*] + p \cdot V^{\mathcal{L}}\right]\right\}, \tag{106}$$

where $W' = Ak^\alpha\varepsilon - Rb$ and the constraint set is:

$$\Gamma(W) = \left\{(k, b) : k \geq 0,\ 0 \leq b \leq \bar{b},\ W + b - k \geq \underline{c}\right\}. \tag{107}$$

D.4.2 Reformulation

It is convenient to rewrite the continuation value as:

$$\mathcal{V}(k, b; A, \sigma, V) = \int_{\varepsilon^*}^{\infty} V(A, Ak^\alpha\varepsilon - Rb, \sigma)g(\varepsilon; \sigma)d\varepsilon + G(\varepsilon^*; \sigma) \cdot V^{\mathcal{L}}. \tag{108}$$

Then:

$$V^U(A,W,\sigma) = \max_{(k,b)\in\Gamma(W)} \left\{u(W+b-k) + \beta\mathcal{V}(k,b;A,\sigma,V^U)\right\}. \tag{109}$$

D.4.3 The Operator

Define $T^U : \mathcal{B}(\mathcal{S}) \to \mathcal{B}(\mathcal{S})$ by:

$$(T^U V)(A,W,\sigma) = \max_{(k,b)\in\Gamma(W)} \left\{u(W+b-k) + \beta\mathcal{V}(k,b;A,\sigma,V)\right\}. \tag{110}$$

Lemma 3 (Constraint Set Properties)**.** *For each $W \in \mathcal{W}$:*

(i) *$\Gamma(W)$ is non-empty: $(k,b) = (0,0) \in \Gamma(W)$ since $W \geq \underline{W} > \underline{c}$.*

(ii) *$\Gamma(W)$ is compact: $k \in [0, W + \bar{b} - \underline{c}]$, $b \in [0, \bar{b}]$.*

(iii) *Γ is continuous (lower and upper hemicontinuous) in W.*

Proof. (i) and (ii) are immediate. (iii): The constraint $W + b - k \geq \underline{c}$ defines a half-space that shifts continuously with W. □

Lemma 4 (Continuation Value Properties)**.** *For any $V \in \mathcal{B}(\mathcal{S})$ with $\|V\|_\infty \leq M$:*

(i) *$\mathcal{V}(k,b;A,\sigma,V)$ is bounded: $|\mathcal{V}| \leq M$.*

(ii) *$\mathcal{V}$ is continuous in (k,b) for fixed (A,σ,V).*

(iii) *$\mathcal{V}$ is Lipschitz in V: $|\mathcal{V}(\cdot;V) - \mathcal{V}(\cdot;V')| \leq \|V - V'\|_\infty$.*

Proof. (i): Both terms are bounded by M.

(ii): The integrand $V(A, Ak^\alpha\varepsilon - Rb, \sigma)$ is continuous in (k,b) for each ε (by boundedness, even though V may not be continuous). The limits of integration ε^* depend continuously on (k,b). By dominated convergence, the integral is continuous.

(iii):

$$\begin{aligned}
|\mathcal{V}(\cdot;V) - \mathcal{V}(\cdot;V')| &\leq \int_{\varepsilon^*}^{\infty} |V - V'| g(\varepsilon) d\varepsilon \qquad (111)\\
&\leq \|V - V'\|_\infty \int_{\varepsilon^*}^{\infty} g(\varepsilon) d\varepsilon \qquad (112)\\
&\leq \|V - V'\|_\infty.
\end{aligned}$$

□

Theorem 8 (Contraction for Unprotected Regime)**.** *The operator $T^{\mathcal{U}} : \mathcal{B}(\mathcal{S}) \to \mathcal{B}(\mathcal{S})$ is a contraction with modulus β:*

$$\|T^{\mathcal{U}}V - T^{\mathcal{U}}V'\|_\infty \leq \beta\|V - V'\|_\infty. \tag{113}$$

Proof. Fix (A, W, σ) and let (k^*, b^*) attain the maximum in $(T^{\mathcal{U}}V)(A, W, \sigma)$. Then:

$$(T^{\mathcal{U}}V)(A, W, \sigma) - (T^{\mathcal{U}}V')(A, W, \sigma) \leq u(W + b^* - k^*) + \beta\mathcal{V}(k^*, b^*; V) \tag{114}$$

$$- u(W + b^* - k^*) - \beta\mathcal{V}(k^*, b^*; V') \tag{115}$$

$$= \beta\left[\mathcal{V}(k^*, b^*; V) - \mathcal{V}(k^*, b^*; V')\right] \tag{116}$$

$$\leq \beta\|V - V'\|_\infty. \tag{117}$$

Symmetrically for the reverse inequality. Taking the supremum completes the proof. □

Theorem 9 (Existence and Uniqueness — Unprotected Regime)**.** *Under Assumptions 2–9, there exists a unique $V^{\mathcal{U}} \in \mathcal{B}(\mathcal{S})$ satisfying the Bellman equation (106). Moreover:*

(i) $V^{\mathcal{U}}$ is the uniform limit of $V_n = T^{\mathcal{U}}V_{n-1}$ from any $V_0 \in \mathcal{B}$.

(ii) $V^{\mathcal{U}}(A, W, \sigma) \geq V^{\mathcal{L}}$ for all (A, W, σ).

(iii) $V^{\mathcal{U}}$ is weakly increasing in A and W, weakly decreasing in σ.

Proof. (i): By Theorem 8 and the Banach fixed point theorem.

(ii): The household can always guarantee at least $V^{\mathcal{L}}$ (by choosing policies that lead to certain default, or simply by noting that even with positive survival probability, the continuation value exceeds $V^{\mathcal{L}}$).

(iii): Monotonicity arguments parallel those in Theorem 7. Higher A increases output and reduces default probability. Higher W expands the constraint set and increases consumption. Higher σ increases default probability, reducing continuation value. □

Remark 3 (Continuity of $V^{\mathcal{U}}$)**.** *Unlike the protected case, $V^{\mathcal{U}}$ may not be continuous everywhere. Discontinuities can arise at points where the optimal policy switches discretely (e.g., from borrowing to not borrowing) or where the default threshold ε^* passes through a mass point of the* *shock distribution. Under the log-normal assumption (continuous density), V^U is continuous almost everywhere. For our purposes, measurability and boundedness suffice.*

D.5 Optimal Policy under Unprotected Tenure

D.5.1 First-Order Conditions

Let (k^U, b^U) denote the optimal policy. At an interior solution, the first-order conditions are:

Proposition 23 (FOCs for Unprotected Regime)**.** *When the solution is interior ($(k^U, b^U) \in \text{int}(\Gamma)$):*

$$u'(c) = \beta \frac{\partial \mathcal{V}}{\partial k}, \tag{118}$$

$$u'(c) = \beta \frac{\partial \mathcal{V}}{\partial b}, \tag{119}$$

where $c = W + b - k$.

The partial derivatives are:

$$\frac{\partial \mathcal{V}}{\partial k} = \alpha A k^{\alpha-1} \int_{\varepsilon^*}^{\infty} V_W^U(A, W', \sigma) \varepsilon \, g(\varepsilon) d\varepsilon \tag{120}$$

$$- \left[V^U(A, \underline{c}, \sigma) - V^{\mathcal{L}} \right] g(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial k}, \tag{121}$$

$$\frac{\partial \mathcal{V}}{\partial b} = -R \int_{\varepsilon^*}^{\infty} V_W^U(A, W', \sigma) g(\varepsilon) d\varepsilon \tag{122}$$

$$- \left[V^U(A, \underline{c}, \sigma) - V^{\mathcal{L}} \right] g(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial b}, \tag{123}$$

where:

$$\frac{\partial \varepsilon^*}{\partial k} = -\frac{\alpha(Rb + \underline{c})}{A k^{\alpha+1}} < 0, \quad \frac{\partial \varepsilon^*}{\partial b} = \frac{R}{A k^{\alpha}} > 0. \tag{124}$$

Proof. Differentiate $\mathcal{V}$ using Leibniz's rule, noting that the integrand is zero at the lower limit $\varepsilon = \varepsilon^*$ (where $W' = \underline{c}$). □

D.5.2 Interpretation

The FOC for k equates the marginal utility of consumption to:

- The direct marginal product of capital (first term): higher k raises output and continuation value.
- The risk reduction benefit (second term): higher k lowers ε^*, reducing default probability.

The FOC for b equates marginal utility to:

- The direct cost of debt service (first term): higher b requires repayment Rb, reducing W'.
- The risk increase cost (second term): higher b raises ε^*, increasing default probability.

D.5.3 When Does the Borrowing Constraint Bind?

Proposition 24 (Borrowing Constraint)**.** *The borrowing constraint $b \leq \bar{b}$ binds if and only if the marginal value of borrowing exceeds its marginal cost at $b = \bar{b}$:*

$$u'(c)|_{b=\bar{b}} > \beta \left| \frac{\partial \mathcal{V}}{\partial b} \right|_{b=\bar{b}}. \tag{125}$$

This occurs when:

(i) A is high (making investment very productive).

(ii) W is low (making the household wealth-constrained).

(iii) σ is low (reducing the risk penalty for borrowing).

D.6 The Conversion Threshold Surface

D.6.1 The Net Benefit of Conversion

Definition 7 (Conversion Gain)**.** *The net benefit of converting from protected to unprotected tenure is:*

$$\Delta(A, W, \sigma) = V^U(A, W, \sigma) - V^P(A, W). \tag{126}$$

A household converts if and only if $\Delta > 0$.

Lemma 5 (Regularity of Δ)**.** *Under the maintained assumptions:*

(i) Δ *is measurable in* (A, W, σ)*.*

(ii) Δ *is continuous in* (A, W) *for each* σ *(where* V^U *is continuous).*

(iii) $\lim_{A\to\infty} \Delta(A, W, \sigma) = +\infty$ *for any fixed* (W, σ) *with* $W > \underline{c}$*.*

(iv) $\Delta(\underline{A}, W, \sigma) < 0$ *for* σ *sufficiently large.*

Proof. (i), (ii): Follow from the corresponding properties of V^U and V^P.

(iii): As $A \to \infty$, the default probability $p \to 0$ (since $\varepsilon^* = \frac{Rb+\underline{c}}{Ak^\alpha} \to 0$). The household can borrow, invest efficiently, and almost surely avoid default. The value $V^U \to \infty$ (or to the efficient frontier), while V^P grows more slowly because it cannot leverage. Hence $\Delta \to +\infty$.

(iv): At $A = \underline{A}$, productivity is low. Output $\underline{A}k^\alpha\varepsilon$ may fall below $Rb + \underline{c}$ with high probability when σ is large. The household gains little from borrowing (low marginal product) but faces high default risk. For σ large enough, the risk cost exceeds the credit benefit, so $\Delta < 0$. □

D.6.2 Existence of the Threshold

Theorem 10 (Threshold Existence)**.** *For each* $(W, \sigma) \in \mathcal{W} \times \Sigma$ *with* $W > \underline{c}$*, there exists* $\bar{A}(W, \sigma) \in (\underline{A}, \bar{A})$ *such that:*

$$\Delta(A, W, \sigma) \gtreqless 0 \iff A \gtreqless \bar{A}(W, \sigma). \tag{127}$$

Proof. Step 1: Boundary values.

By Lemma 5(iv), for σ large, $\Delta(\underline{A}, W, \sigma) < 0$.

By Lemma 5(iii), $\Delta(\bar{A}, W, \sigma) > 0$ for $\bar{A}$ large enough (we assume $\bar{A}$ is chosen to satisfy this).

Step 2: Monotonicity in A.

We show Δ is strictly increasing in A. Both V^U and V^P are increasing in A, but V^U increases faster.

Under protected tenure, higher A raises output proportionally: $V^P(A', W) - V^P(A, W) \approx \frac{\partial V^P}{\partial A}(A' - A)$.

Under unprotected tenure, higher A raises output and also reduces default probability:

$$\frac{\partial V^U}{\partial A} = \left.\frac{\partial V^U}{\partial A}\right|_{\text{direct}} + \frac{\partial V^U}{\partial p} \cdot \frac{\partial p}{\partial A}. \tag{128}$$

Since $\frac{\partial p}{\partial A} < 0$ and $\frac{\partial V^U}{\partial p} < 0$, the second term is positive. Hence:

$$\frac{\partial \Delta}{\partial A} = \frac{\partial V^U}{\partial A} - \frac{\partial V^P}{\partial A} > 0. \tag{129}$$

Step 3: Intermediate Value Theorem.

Since $\Delta(\underline{A}, \cdot) < 0$, $\Delta(\bar{A}, \cdot) > 0$, and Δ is continuous and strictly increasing in A, there exists a unique $\bar{A}(W, \sigma)$ such that $\Delta(\bar{A}, W, \sigma) = 0$. □

D.6.3 Properties of the Threshold

Proposition 25 (Threshold Comparative Statics)**.** *The threshold function $\bar{A} : \mathcal{W} \times \Sigma \to \mathcal{A}$ satisfies:*

(i) $\frac{\partial \bar{A}}{\partial \sigma} > 0$*: higher risk requires higher productivity to justify conversion.*

(ii) The sign of $\frac{\partial \bar{A}}{\partial W}$ *is ambiguous in general.*

Proof. By the implicit function theorem applied to $\Delta(\bar{A}(W, \sigma), W, \sigma) = 0$:

$$\frac{\partial \bar{A}}{\partial \sigma} = -\frac{\partial \Delta / \partial \sigma}{\partial \Delta / \partial A}. \tag{130}$$

Since $\partial \Delta / \partial A > 0$ (proved above) and $\partial \Delta / \partial \sigma < 0$ (higher σ increases default risk, reducing V^U while leaving V^P unchanged):

$$\frac{\partial \bar{A}}{\partial \sigma} = -\frac{(-)}{(+)} > 0. \tag{131}$$

For $\frac{\partial \bar{A}}{\partial W}$:

$$\frac{\partial \bar{A}}{\partial W} = -\frac{\partial \Delta / \partial W}{\partial \Delta / \partial A}. \tag{132}$$

The sign of $\partial\Delta/\partial W = \partial V^U/\partial W - \partial V^P/\partial W$ is ambiguous:

- Higher W relaxes the credit constraint under protection, raising V^P.
- Higher W provides a buffer under unprotected tenure, raising V^U.

The net effect depends on which is more binding. □

D.6.4 The Threshold Surface

The threshold defines a surface in (A, W, σ) space:

$$\mathcal{T} = \{(A, W, \sigma) \in \mathcal{S} : \Delta(A, W, \sigma) = 0\} = \{(A, W, \sigma) : A = \bar{A}(W, \sigma)\}. \tag{133}$$

The conversion region is:

$$\mathcal{C} = \{(A, W, \sigma) \in \mathcal{S} : A > \bar{A}(W, \sigma)\}. \tag{134}$$

D.7 Infinite-Horizon Dynamics

We now analyze the evolution of the population distribution over time.

D.7.1 Probability Measures and Transition Kernels

Let $\mathcal{P}(\mathcal{S})$ denote the space of Borel probability measures on $\mathcal{S}$. Let $\mu_t \in \mathcal{P}(\mathcal{S} \cup \{\mathcal{L}\})$ denote the distribution of households at time t, where $\mathcal{L}$ is the absorbing "landless" state.

Decompose:

$$\mu_t^P = \text{measure on } \mathcal{S} \text{ of protected households,} \tag{135}$$

$$\mu_t^U = \text{measure on } \mathcal{S} \text{ of unprotected households (not yet defaulted),} \tag{136}$$

$$\mu_t^{\mathcal{L}} = \text{mass at } \mathcal{L} \text{ (landless households).} \tag{137}$$

D.7.2 Initial Conditions

At $t = 0$, households are distributed according to F on $\mathcal{S}$. Each chooses whether to convert:

$$\mu_0^P = F|_{\mathcal{S}\setminus\mathcal{C}} = \int_{\mathcal{S}\setminus\mathcal{C}} dF, \tag{138}$$

$$\mu_0^U = F|_{\mathcal{C}} = \int_{\mathcal{C}} dF, \tag{139}$$

$$\mu_0^{\mathcal{L}} = 0. \tag{140}$$

Let $\lambda = \mu_0^U(\mathcal{S}) = F(\mathcal{C})$ denote the conversion rate.

D.7.3 Transition Dynamics

For $t \geq 1$:

Protected households: The type (A, σ) is fixed; only wealth W evolves. The transition kernel is:

$$Q^P((A, W, \sigma), B) = \mathbb{P}(W' \in B_W | A, W, \sigma) = \int \mathbf{1}_{B_W}(A(W - k^P)^\alpha \varepsilon) g(\varepsilon) d\varepsilon, \tag{141}$$

where $B_W = \{w : (A, w, \sigma) \in B\}$.

Unprotected households: With probability $1 - p(k^U, b^U; A, \sigma)$, the household survives and wealth evolves. With probability p, the household defaults and transitions to $\mathcal{L}$.

$$Q^U((A, W, \sigma), B) = (1 - p) \cdot \mathbb{P}(W' \in B_W | \text{no default}) + p \cdot \mathbf{1}_{\mathcal{L}}(B). \tag{142}$$

D.7.4 Evolution Equations

The measures evolve as:

$$\mu_{t+1}^{P}(B) = \int_{\mathcal{S}} Q^{P}(s,B)\, d\mu_t^{P}(s), \tag{143}$$

$$\mu_{t+1}^{U}(B) = \int_{\mathcal{S}} (1-p(s))Q_{\text{survive}}^{U}(s,B)\, d\mu_t^{U}(s), \tag{144}$$

$$\mu_{t+1}^{\mathcal{L}} = \mu_t^{\mathcal{L}} + \int_{\mathcal{S}} p(s)\, d\mu_t^{U}(s). \tag{145}$$

D.7.5 The Ratchet Theorem

Theorem 11 (Exponential Decay of Unprotected Land)**.** *Suppose $p(s) \geq \underline{p} > 0$ for all $s \in \operatorname{supp}(\mu_0^U)$. Then:*

(i) $\mu_t^U(\mathcal{S}) \leq \lambda(1-\underline{p})^t$.

(ii) $\lim_{t\to\infty} \mu_t^U(\mathcal{S}) = 0$.

(iii) $\lim_{t\to\infty} \mu_t^{\mathcal{L}} = \lambda$.

Proof. (i): Let $m_t = \mu_t^U(\mathcal{S})$ denote the total mass of unprotected households at time t. From the evolution equation:

$$m_{t+1} = \int_{\mathcal{S}} (1-p(s))\, d\mu_t^U(s) \tag{146}$$

$$\leq (1-\underline{p}) \int_{\mathcal{S}} d\mu_t^U(s) \tag{147}$$

$$= (1-\underline{p})m_t. \tag{148}$$

By induction: $m_t \leq m_0(1-\underline{p})^t = \lambda(1-\underline{p})^t$.

(ii): Since $\underline{p} > 0$, $(1-\underline{p}) < 1$, so $(1-\underline{p})^t \to 0$.

(iii): Conservation of mass: $\mu_t^P(\mathcal{S}) + m_t + \mu_t^{\mathcal{L}} = 1$. Since $\mu_t^P(\mathcal{S}) = 1-\lambda$ (constant) and $m_t \to 0$, we have $\mu_t^{\mathcal{L}} \to \lambda$. □

Corollary 7 (Half-Life)**.** *The half-life of unprotected land (time until $m_t = \frac{\lambda}{2}$) is:*

$$t_{1/2} = \frac{\ln 2}{-\ln(1-\underline{p})} \approx \frac{\ln 2}{\underline{p}} \quad \textit{for small } \underline{p}. \tag{149}$$

Proof. Solve $\lambda(1-\underline{p})^{t_{1/2}} = \frac{\lambda}{2}$:

$$t_{1/2} = \frac{\ln(1/2)}{\ln(1-\underline{p})} = \frac{\ln 2}{-\ln(1-\underline{p})}. \tag{150}$$

For small $\underline{p}$, $-\ln(1-\underline{p}) \approx \underline{p}$. □

D.7.6 Selection Effects

Proposition 26 (Adverse Selection). *Suppose the conversion region $\mathcal{C}$ overweights high-σ households relative to the population F:*

$$\mathbb{E}[\sigma|\mathcal{C}] > \mathbb{E}[\sigma]. \tag{151}$$

Then the average default probability among converters exceeds the population average:

$$\bar{p} = \mathbb{E}[p(s)|s \in \mathcal{C}] > \mathbb{E}[p(s)]. \tag{152}$$

The ratchet operates faster than it would under random conversion.

Proof. Since $\frac{\partial p}{\partial \sigma} > 0$, higher average σ implies higher average p. □

Proposition 27 (Selection Over Time). *As t increases, the surviving unprotected population μ_t^U becomes increasingly selected toward low-p types. However, if $\underline{p} > 0$ for all types, selection cannot halt the ratchet—it only slows it.*

Proof. Households with higher p exit faster (through default), so survivors have lower average p. But as long as $\underline{p} > 0$, even the lowest-p types eventually default. □

D.7.7 Correlated Defaults

Proposition 28 (Aggregate Shocks). *Conditional on the aggregate shock η_t, the default rate is:*

$$D_t(\eta_t) = \int_{\mathcal{S}} p(s|\eta_t)\, d\mu_t^U(s), \tag{153}$$

where $p(s|\eta_t) = \Phi\left(\frac{\ln \varepsilon^*(s) + \frac{1}{2}\sigma_i^2 - \eta_t}{\sigma_i}\right)$ is the conditional default probability.

Bad aggregate shocks (low η_t) increase D_t for all households simultaneously, generating correlated defaults.

Corollary 8 (Variance of Defaults)**.** *The unconditional variance of aggregate defaults is:*

$$\text{Var}(D_t) = \mathbb{E}[\text{Var}(D_t|\eta_t)] + \text{Var}(\mathbb{E}[D_t|\eta_t]). \tag{154}$$

The second term captures systematic risk from aggregate shocks. It is positive whenever $\sigma_\eta > 0$.

D.8 Extension: Insurance

D.8.1 The Insurance Contract

Definition 8 (Full Default Insurance)**.** *An insurance contract pays the shortfall when income falls below the default threshold:*

$$I(\varepsilon) = \max\{0, Rb + \underline{c} - Ak^\alpha \varepsilon\} = (Rb + \underline{c} - Ak^\alpha \varepsilon)^+. \tag{155}$$

The actuarially fair premium is:

$$\pi^*(k, b; A, \sigma) = \mathbb{E}[I(\varepsilon)] = \int_0^{\varepsilon^*} (Rb + \underline{c} - Ak^\alpha \varepsilon) g(\varepsilon; \sigma) d\varepsilon. \tag{156}$$

Lemma 6 (Premium Properties)**.** *The fair premium satisfies:*

(i) $\pi^* \geq 0$, *with equality iff* $p = 0$.

(ii) $\frac{\partial \pi^*}{\partial b} > 0$, $\frac{\partial \pi^*}{\partial \sigma} > 0$, $\frac{\partial \pi^*}{\partial A} < 0$, $\frac{\partial \pi^*}{\partial k} < 0$.

D.8.2 Value Function with Insurance

With mandatory insurance at fair premium:

$$V^{U,I}(A, W, \sigma) = \max_{(k,b) \in \Gamma^I(W)} \left\{ u(W + b - k - \pi^*(k, b)) + \beta \mathbb{E}[V^{U,I}(A, W', \sigma)] \right\}, \tag{157}$$

where $W' = Ak^\alpha \varepsilon - Rb + I(\varepsilon) \geq \underline{c}$ always (no default).

Theorem 12 (No Default with Insurance)**.** *Under full insurance at fair premium, $p^I = 0$. The ratchet is eliminated: $\mu_t^U(\mathcal{S}) = \lambda$ for all t.*

Proof. By construction, $I(\varepsilon)$ covers any shortfall, so $W' \geq \underline{c}$ always. No household defaults, so μ_t^U is preserved. □

D.8.3 Insurance Changes the Threshold

Proposition 29 (Higher Threshold with Insurance)**.** *Let $\bar{A}^I(W, \sigma)$ be the conversion threshold with mandatory insurance. Then:*

$$\bar{A}^I(W, \sigma) > \bar{A}(W, \sigma) \quad \forall (W, \sigma). \tag{158}$$

Proof. The premium π^* is a cost that reduces the net benefit of conversion. For a marginal converter ($\Delta = 0$), the premium cost pushes $\Delta^I = V^{U,I} - V^P < 0$. The threshold must rise to compensate. □

Corollary 9 (Selection with Insurance)**.** *Under mandatory insurance:*

(i) Fewer households convert: $\lambda^I < \lambda$.

(ii) Converting households have higher average A (positive selection).

(iii) The ratchet is eliminated.

D.9 Summary of Results

1. **Existence and Uniqueness (Theorems 7, 9):** Value functions V^P and V^U exist, are unique, bounded, and satisfy standard monotonicity properties. V^P is continuous and concave in wealth; V^U is measurable and monotone.

2. **Optimal Policies (Propositions 20, 23):** Policies are characterized by Euler equations with additional terms under unprotected tenure reflecting the marginal impact of choices on default probability.

3. **Conversion Threshold (Theorem 10, Proposition 25):** A threshold surface $\bar{A}(W, \sigma)$ separates converters from non-converters. The threshold is increasing in risk σ.

4. **Ratchet Dynamics (Theorem 11):** Under any regime with positive default probability, land held by converters decays exponentially to zero. Adverse selection accelerates the ratchet (Proposition 26).

5. **Insurance (Theorem 12, Proposition 29):** Mandatory insurance eliminates default, raises the conversion threshold (filtering out marginal converters), and stops the ratchet.

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